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ABSTRACT

A simplified analytical model for two-phase subcritical flow in orifices is provided. Critical choking conditions are obtained as a limit case. Using one-dimensional separated flow equations, the downstream flow is shown to depend on stagnation properties and on an overall friction coefficient characterizing the orifice geometry. The model allows for thermal non-equilibrium and for relative motion between the two phases. The results are readily applicable as design tools.

NOMENCLATURE

c_l, c_p specific heat at constant pressure, J/Kg/K
D orifice diameter, m
$\dot{E}$ Energy dissipation, J/s
F Fanning friction factor, -
G Mass flow rate per unit area, $Kg/m^2/s$
K Gas to liquid velocity ratio, -
R perfect gas constant, J/mole/K
p pressyrem B.m/^{2}
T temperature, K
w velocity, m/s
x quality, -
z axial length, m
α void fraction, -
γ specific heat ratio of gas phase, -
ρ density, Kg/m^3
ξ DarcY-Weisbech friction coefficient, -
ϕ^2 Two-phase multiplier, -

Subscripts

c choking conditions
g gas phase
ℓ liquid phase
o stagnation condition

INTRODUCTION

We present a simplified analytical model for two-phase subcritical flow in orifices. The model yields relations between the up and downstream flow properties and the overall friction coefficient generally used to characterize orifice geometry. The results are readily applicable to practical nozzle design. The model extends all the way to critical choking conditions.

The two-phase critical flow in nozzles has been the subject of many analytical and experimental investigations [1]. The physical models used in these studies have been derived either from the work by Moody [2-4] based on the assumption that the two phases are in mutual thermodynamic equilibrium, or from the work by Fauske [5-7] based on the assumption that no phase change occurs.

Here, we adopt the one-dimensional separated flow model, with the assumption of no phase change and extend the treatment to all conditions, including the critical flow as a limit case. Our assumptions include that the friction losses from the mixture are due to the liquid phase only, so that the gas phase behaves as an ideal gas undergoing an isentropic expansion. Moreover, the two phases can have different temperatures, consistently with the general ideas of the constrained-equilibrium method to deal with the description of nonequilibrium states [8].

ASSUMPTIONS

The basic assumptions of the model can be summarized as follows:

a. The quality x, i.e., the gas fraction of the total mass flow rate, is constant along the length of the orifice, i.e.,

$$dx/dz = 0 \tag{1}$$

where z is the axial coordinate. In other words, we assume that no appreciable phase change occurs in the orifice -- like, e.g., for a water-air mixture sufficiently far from saturation.

b. Both the liquid phase and the gas phase are in thermodynamic equilibrium at temperatures T_ℓ and T_g, respectively, but T_ℓ and T_g are not necessarily equal, i.e., the two phases are not necessarily in mutual equilibrium.

c. The gas phase behaves as a perfect gas, so that

$$p = \rho_g R T_g \tag{2}$$

d. The gas phase undergoes an isentropic expansion, so that

$$\frac{1}{p}\frac{dp}{dz} = \gamma \frac{1}{\rho_g}\frac{d\rho_g}{dz} \tag{3}$$

where γ is the specific heat ratio. This hypothesis stems from the fact that the friction of the gas phase, both against the walls and at the gas-liquid interface, is neglected.

e. The liquid phase behaves as an incompressible fluid, so that

$$\frac{d\rho_\ell}{dz} = 0 \tag{4}$$

f. The slip ratio κ, i.e., the ratio of the gas speed w_g to the liquid speed w_ℓ, is related to the density ratio ρ_g/ρ_ℓ by

$$\frac{w_g}{w_\ell} = \kappa = \left(\frac{\rho_\ell}{\rho_g}\right)^r \tag{5}$$

As discussed by Zivi [9] the value $r = 1/3$ corresponds to maximum rate of entropy production or maximum possible discharge rate for a given nozzle exit area (Wallis [10]). However, for air-water flows, Fauske [5] observed slip ratios much closer to unity.

g. The flow is choked when the flow properties show a discontinuity in the axial coordinate z.

h. The balance equations for mass, momentum and energy are written under the assumption of steady one-dimensional flow.

BALANCE EQUATIONS

The mass balance equation can be written as

$$\frac{dG}{dz} = \frac{d}{dz}(\rho_g w_g \alpha / x) = 0 \tag{6}$$

where G is the total mass flow rate per unit of cross sectional area, and α is the void fraction given by the relation

$$\alpha = \left[1 + \frac{1-x}{x}\,\kappa\,\frac{\rho_g}{\rho_\ell}\right]^{-1} \tag{7}$$

The momentum balance equation can be written as

$$\frac{dp}{dz} + \frac{d}{dz}[G_\ell w_\ell + G_g w_g] = \left(\frac{dp}{dz}\right)_f \tag{8}$$

where $G_\ell = (1-x)G = \rho_\ell w_\ell (1-\alpha)$ is the liquid flow rate per unit area, $G_g = xG = \rho_g w_g \alpha$ is the gas flow rate per unit area, and $-(dp/dz)_f$ is the rate of frictional pressure drop. We further assume that

$$\left(\frac{dp}{dz}\right)_f = -\frac{2 f_\ell G^2 \phi^2}{\rho_\ell D} \tag{9}$$

where f_ℓ is the Fanning friction factor for liquid flow, D is the orifice diameter, and ϕ^2 is Martinelli's correction factor taking into account the two-phase effects (Wallis [10]).

The energy balance equation can be written as

$$\frac{d}{dz}\left[G_g\left(c_{pg}T_g + \frac{1}{2}w_g^2\right) + G_\ell\left(c_\ell T_\ell + \frac{1}{2}w_\ell^2\right)\right] = \dot{E} \tag{10}$$

where c_{pg} is the specific heat at constant pressure of the gas phase, c_ℓ the specific heat of the liquid phase, and $\dot{E}$ represents the energy transfer rate per unit wall area. Equation 10 will not be considered in the following, since it adds to the problem a new variable, i.e., T_ℓ, which is not of great interest to us.

We now differentiate Equation 5 to obtain

$$\frac{dw_g}{dz} = \kappa \frac{dw_\ell}{dz} - r \frac{w_g}{\rho_g}\frac{d\rho_g}{dz} \tag{11}$$

and Equation 2 to obtain

$$\frac{dp}{dz} = RT_g \frac{d\rho_g}{dz} + R\rho_g \frac{dT_g}{dz} \tag{12}$$

SUMMARY OF MODEL EQUATIONS

Equations 3, 6, 8, 9, 11, 12 can be combined to yield a system of linear equations in the unknowns dp/dz, dw_ℓ/dz, $d\rho_g/dz$, dT_g/dz that can be written as

$$[A][Y] = [B] \tag{13}$$

where

$$[Y]^T = [dp/dz, dw_\ell/dz, d\rho_g/dz, dT_g/dz] \tag{14}$$

$$[B]^T = [0, 0, -2f_\ell G^2\phi^2/\rho_\ell D, 0] \tag{15}$$

$$[A] = \begin{bmatrix} \rho_g & 0 & -\gamma p & 0 \\ 0 & \rho_g \kappa & \alpha\kappa w_\ell (1-r) & 0 \\ 1 & G(1+x\kappa-x) & -Gxr\kappa w_\ell/\rho_g & 0 \\ 1 & 0 & -RT_g & -R\rho_g \end{bmatrix} \tag{16}$$

SOLUTION

The critical or choking conditions, according to assumption (g), are defined by a discontinuity in the flow properties. The discontinuity occurs when the determinant of matrix [A] vanishes, i.e., when

$$[(1+x\kappa_c-x)(1-r)x + x^2 r\kappa_c/\alpha_c]G_c^2 - \gamma p_c \rho_{gc}\kappa_c = 0 \tag{17}$$

where subscript "c" denotes choking conditions.

Under the assumptions adopted here, for a given constant value of x and ρ_ℓ, the values of κ and α depend on ρ_g only, and so does the dimensionless factor $F(\rho_g)$ defined by the relation

$$F(\rho_g) = (1 + x\kappa - x)(1 - r)x/\kappa + x^2 r/\alpha \tag{18}$$

In terms of this factor, the critical or choking flow rate becomes (Equations 17 and 18)

$$G_c = [\gamma p_c \rho_{gc}/F(\rho_{gc})]^{1/2} \tag{19}$$

For homogeneous flow ($\kappa = 1$, $r = 0$) the critical mass flow rate reduces to its homogeneous value

$$G_c = [\gamma p_c \rho_{gc}/x]^{1/2} \tag{20}$$

a result identical to Moody's expression [4] for an incompressible liquid.

For the flow conditions from the inlet of the orifice to the choking location the system of Equations 13 can be solved, e.g., by Kramer's rule, to yield

$$\frac{d\rho_g}{dz} = \frac{2f_\ell G^2 \phi^2}{\rho_\ell D} \frac{\rho_g^2}{F(\rho_g)G^2 - \gamma p \rho_g} \tag{21}$$

Equation 21 may be combined with Equation 19 and $p/p_c = (\rho_g/\rho_{gc})^\gamma$ (Equation 3) to yield the relation

$$\frac{2f_\ell \phi^2}{\rho_\ell D} dz = [F(\rho_g) - (\frac{G_c}{G})^2 F(\rho_{gc})(\frac{\rho_g}{\rho_{gc}})^{\gamma+1}] \frac{1}{\rho_g^2} d\rho_g \tag{22}$$

Now, for short tubes and orifices, the frictional pressure drop is generally given in terms of the Darcy-Weisbech parameter (Wallis [10])

$$\zeta = \frac{2\bar{\rho}_m (\Delta p)_f}{G^2} \tag{23}$$

where the mean fluid density $\bar{\rho}_m$ is given by

$$\bar{\rho}_m = \alpha\rho_g + (1-\alpha)\rho_\ell \tag{24}$$

Thus, integrating Equation 9 to find $(\Delta p)_f$ yields

$$\zeta = \frac{4f_\ell \phi^2}{\rho_\ell D} \bar{\rho}_m L \tag{25}$$

Inserting this result into Equation 22 and integrating along the axial coordinate of the orifice, we obtain

$$\zeta = 2\bar{\rho}_m \int_{\rho_{go}}^{\rho_{gL}} [F(\rho_g) - (\frac{G_c}{G})^2 F(\rho_{gc})(\frac{\rho_g}{\rho_{gc}})^{\gamma+1}] \frac{1}{\rho_g^2} d\rho_g \tag{26}$$

where the subscripts "o" and "L" refer to inlet and outlet axial location, respectively.

Our calculation method proceeds as follows:

1. For given values of the stagnation properties and of the nozzle geometric parameter ζ, the critical conditions $G = G_c$ and $\rho_{gL} = \rho_{gc}$ are derived from Equations 19 and 26.
2. For given value of the mass flow rate G, the downstream gas density ρ_{gL} is evaluated via Equation 26.
3. The downstream pressure and gas temperature are found via Equations 2 and 3.

RESULTS AND DISCUSSION

When friction at the walls is neglected and no slip is assumed between two phases, i.e., for $\zeta = r = 0$ our model reduces to a simplified Moody's [4] analysis in which the gas is assumed ideal and the liquid incompressible. Thus, it is not surprising that for not too high pressures we obtain the same results as Moody's, which agree with most steam-water critical flow data.

Since the originality of our work is to account for frictional losses, we present here only the predictions where the friction coefficient is non-zero. In particular, results obtained with the homogeneous model can be represented in a dimensionless form based on stagnation properties, much in the same way as for one-phase flow (Shapiro

[11]). Figures 1 and 2 show the dimensionless critical flow rate and the ratio of critical to stagnation pressure as functions of the quality for various loss coefficients. In particular, the critical pressure is nearly constant everywhere except for very low and very high quality, where it decreases, due to the sharp variation of the two-phase critical velocity in those regions.

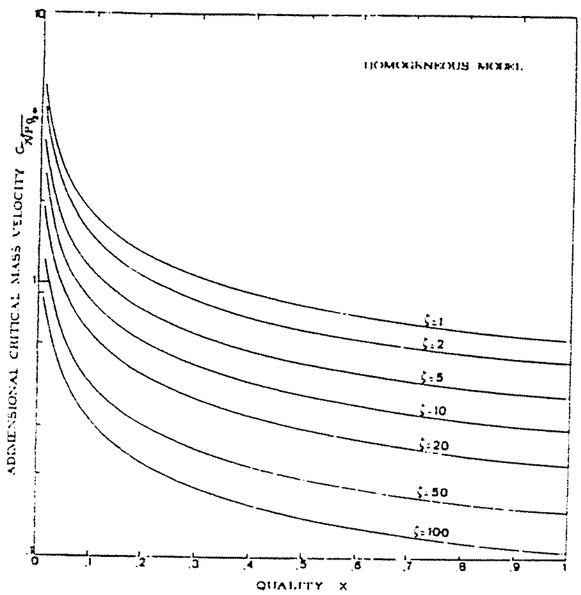

FIG. 1 Dimensionless critical mass velocity versus quality.

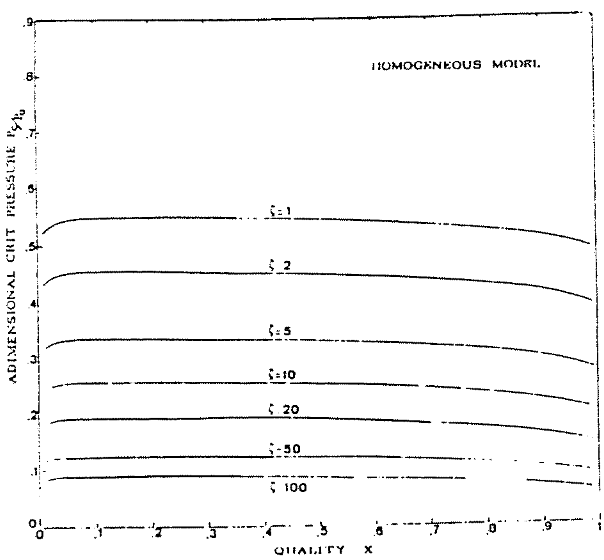

FIG. 2 Ratio of critical to stagnation pressure versus quality.

Figure 3 shows the mass flow rate versus the ratio of exhaust to supply pressure as functions of the quality of the mixture and the equivalent friction coefficient of the nozzle. As it is well known, a reduction in back pressure acts to increase the flow rate until critical conditions (i.e., the dotted line) are reached. Further reduction in back pressure cannot produce further increase in mass flow rate, which means that disturbances initiated downstream of the critical section cannot propagate upstream.

The results of Figure 3 are readily applicable as design tools. For example, when the quality of the mixture is given, the nozzle can be chosen so that the flow conditions are sufficiently far from the critical regime. Again, when the nozzle characteristics are given, the mass flow rate can be determined for any given pressure drop and mixture quality.

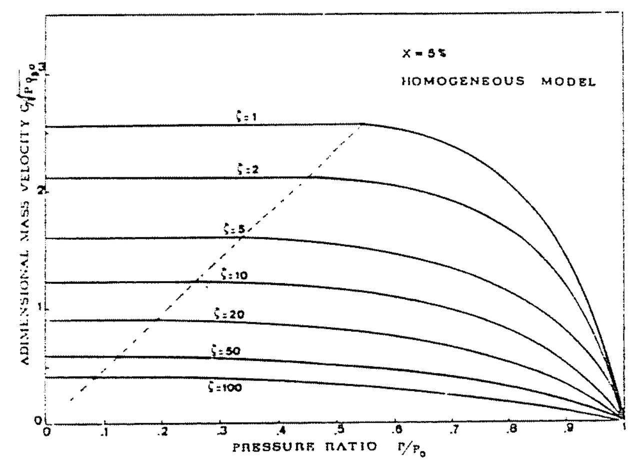

FIG. 3-a

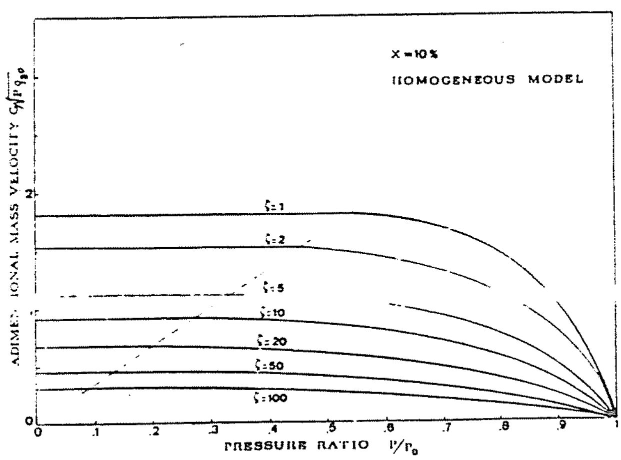

FIG. 3-b

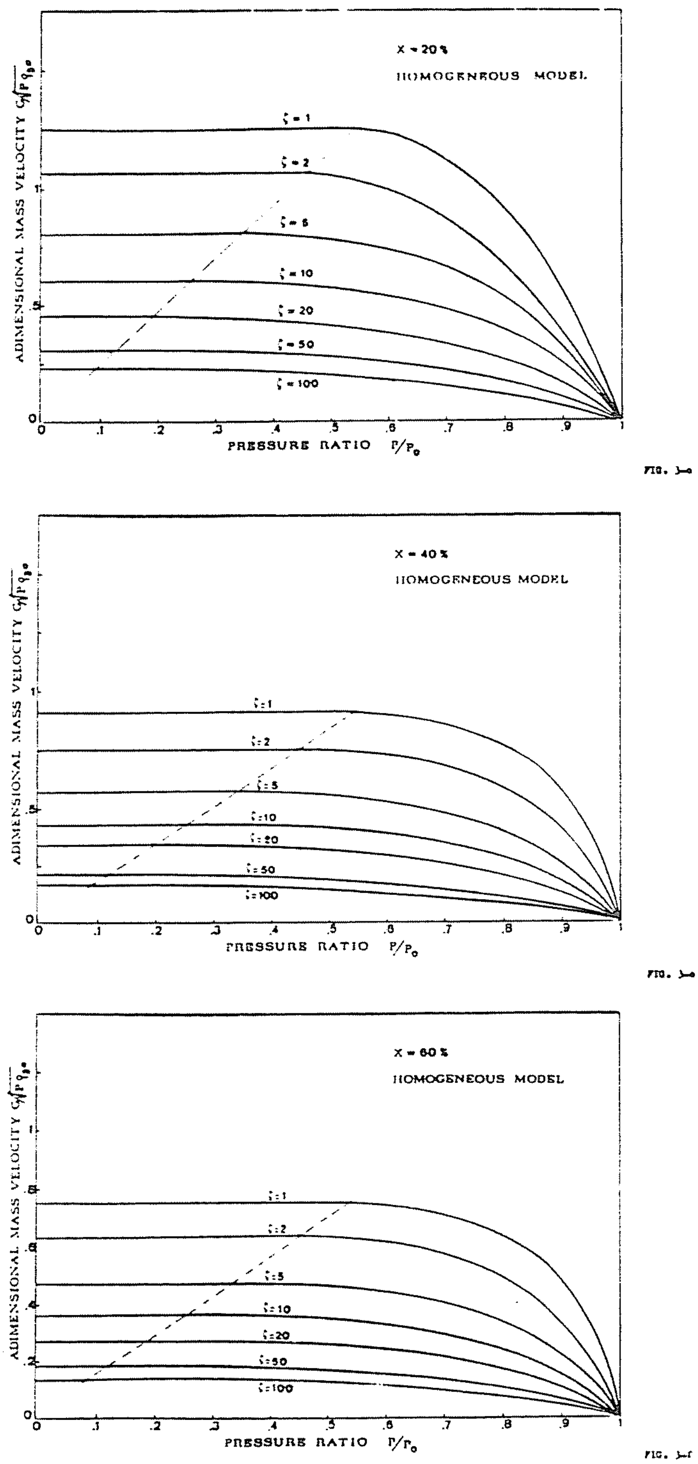

FIG. 3 Dimensionless flow parameter versus ratio of exhaust to supply pressure.

The limitations of our approach should be kept in mind - in particular:

a) Since the gas is assumed to be ideal, our results are applicable to certain two-phase flow only, for example air-water or low pressure steam-water mixtures.

b) Since we assume that the pressure drop is due to the liquid phase only, the title x cannot be too high.

c) Results are given for homogeneous flows only. This is due to the experimental evidence [5] that for short tubes the slip ratio is close to 1 and to the fact that the homogeneous model gives more conservative results. However, we recognize that further investigations are required to clarify this point.

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