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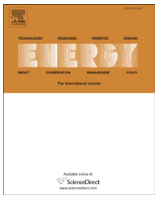

Allocating resources and products in multi-hybrid multi-cogeneration: What fractions of heat and power are renewable in hybrid fossil-solar CHP?

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ABSTRACT

A general method for the allocation of resources and products in multi-resource/multi-product facilities is developed with particular reference to the important two-resource/two-product case of hybrid fossil & solar/heat & power cogeneration. For a realistic case study, we show how the method allows to assess what fractions of the power and heat should be considered as produced from the solar resource and hence identified as renewable. In the present scenario where the hybridization of fossil power plants by solar-integration is gaining increasing attention, such assessment is of great importance in the fair and balanced development of local energy policies based on granting incentives to renewables resources.

The paper extends to the case of two-resource/two-product hybrid cogeneration, as well as to general multi-resource/multi-generation, three of the allocation methods already available for single-resource/two-product cogeneration and for two-resource/single-product hybrid facilities, namely, the ExRR (Exergy-based Reversible-Reference) method, the SRSPR (Single Resource Separate Production Reference) method, and the STALPR (Self-Tuned-Average-Local-Productions-Reference) method. For the case study considered we show that, unless the SRSPR reference efficiencies are constantly updated, the differences between the STALPR and SRSPR methods become important as hybrid and cogeneration plants take up large shares of the local energy production portfolio.

1. Introduction

Many industrial processes can achieve higher production efficiency from the integration of production lines of a mix of different goods and/or using a mix of different resources and/or raw materials. Such facilities may be classified as either multi-resource or multi-generation, or both. Typical examples are the combined production of heat and power [1], or the production of a mix of liquid or gaseous fuels like methanol and hydrogen from a fossil resource, such as coal or natural gas [2]. Likewise, one can combine in the same process multiple resources either fossil or renewable or both, such as coal, natural gas, biomass, geothermal and solar energy, to produce electricity, water desalination, heating, cooling and other by-products [3]. Hybridization of power plants by replacing part of the fossil resources with renewables is currently viewed as a prominent method for future more sustainable scenarios [4]. In order to design regulations with the objective of stimulating these technologies and to promote the use of renewable resources it is important to award incentives and compare efficiencies on the basis of a fair allocation, within each power plant, of the benefits of hybridization and cogeneration.

Several allocation methods have been proposed in the recent years. Broadly speaking they can be divided into thermodynamic methods and thermoeconomic (or exergoeconomic) methods. A comprehensive set of reviews of the rationale of such allocation schemes and their relevant implications can be found with reference to assessing primary energy savings [5] and carbon dioxide emissions [6] as well as in the context of combined production power and desalination [7]. The thermodynamic allocation methods are based on the computation of input/output energy, entropy [8] or exergy fluxes associated to resources and products, generally by comparison with a set of conversion efficiencies, or

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primary energy factors,[1] assumed as references for each resource/product pair. On the other hand, the thermoeconomic methods [9] are based on the exergetic cost balance between input and output exergy streams. They include exergetic cost theory [10], functional approach [11], and specific-cost exergy-costing approach [12].

In previous papers on allocation methods, we addressed two particular cases of multi-resource multi-generation. In Ref. [13] we address the allocation problem in combined heat and power production (i.e., for single-resource/multi-generation facilities), by focusing on the problem of defining a ‘fair’ method to assess how much of the fuel consumption should be allocated to the production of heat and how much to the production of electricity. In such cases, the problem is how to determine the partial efficiencies (or equivalently the primary energy factors) of heat and electricity production in a cogeneration facility.

The work in Ref. [13] was motivated by the fact that none of the existing allocation approaches is yet universally accepted, thus leading to some arbitrariness in the quantification of the economic/energetic/environmental value of the different cogenerated goods and consequently of the basis on which subsidies are awarded to such facilities. A drawback of the classical allocation approaches consists of the adoption of reference efficiencies, determined by some local authority or the regulation itself, for the separate production of heat and electricity that may be different from the actual average efficiencies of heat and electricity in the energy portfolio of the local area of interest with which the cogeneration facility is meant to be compared. Therefore, in Ref. [13] we formulate and discuss (see also Ref. [14]) a novel STALPR (Self-Tuned-Average-Local-Productions-Reference) allocation method whereby the heat and electricity allocation fractions are based on the average efficiencies of the local area (or a prescribed comparison context) that includes the cogeneration plant itself.

In Ref. [15] we address the conjugate allocation problem, i.e., that for multiple-resources/single-product facilities. We focus on hybrid fossil-solar power plants and discuss the problem of defining a ‘fair’ method to allocate fractions of the produced electricity to the different resources consumed. The analysis in Ref. [15] refers to hybrid fossil-solar facilities because they represent a favorable solution in terms of a reliable use of solar energy, capable to mitigate in part its drawbacks due to the intrinsic high degree of daily and seasonal variability. For an overview of different hybrid designs see Ref. [17], for a discussion on the economic feasibility of a typical hybrid configuration see Ref. [18], and for an assessment of the potential of hybrid fossil-solar thermal plants from the point of view of energy policy making see Ref. [19]. In particular, we address the question of what fraction of the electricity produced in such facilities is to be considered as generated from renewables (see also Ref. [16]). In Ref. [15], we extend to this context the definition of the classical SRSPR (Single-Resource-Separate-Production-Reference allocation method) based on reference partial efficiencies prescribed by some authority and we show that it is ‘unfair’ inasmuch as the reference efficiencies are different from the actual average efficiencies of the energy portfolio in local area with which the hybrid facility is to be compared. Thus, also for this context we propose a STLAPR (Self-Tuned-Average-Local-Productions-Reference allocation method) whereby the allocation fractions are defined by the SRSPR method based on reference partial efficiencies identical to the actual average efficiencies that characterize the actual energy production portfolio (including the hybrid plant itself) of the local area of interest for the allocation.

The present paper extends and generalizes the methods of Refs. [13,15] to address the allocation problem for the most general case of multi-resource/multi-generation and exemplifies its application to the relevant case of hybrid fossil-solar combined heat and power production.

We consider our approach a contribution towards a 'fairer' allocation with respect to the existing procedures. Our idea stems from the fact that generation facilities are always part of a local production portfolio. Thus, the overall environmental and economic advantages obtained by introducing a combined/renewable production plant in a certain local area, have a relative impact which depends on the existing local situation and we believe that it is unfair to compute the allocation without taking this into proper account.

The paper is organized as follows. In Section 2, we formally introduce the allocation problem and define the key parameters for the simplest case of a two-resources/two-products (fossil-renewable/heat-electricity) hybrid cogenerator. In Section 3, we define a realistic case study based on hybrid solar-integrated heat and power cogeneration facilities that we use to compare the various allocation methods. In Section 4, we discuss the allocation resulting from the logic of two ‘classical’ methods usually adopted or under consideration by regulators, namely the SRSPR (Single-Resource Separate-Production Reference) method and the ExRR (Exergy-based Reversible-Reference) method. In Section 5, we introduce our context-dependent adaptive marginal allocation method (that we call STALPR). In Section 6, we define the PES (Primary Energy Savings) parameters characterizing a hybrid cogenerator. In Section 7, we formalize the extension of the SRSPR and STALPR methods to the general case of multi-resource/multi-generation. In Section 8, we draw our conclusions. In the Appendix, we provide formulas that simplify the computation of STALPR allocation fractions for two-resources/two-products cases.

2. Allocation problem definition for a hybrid renewable-and-fossil-fuel heat and power cogenerator

In this section we consider a hybrid facility which is also a cogeneration plant, producing on a yearly basis an amount W_{hyb} of electrical energy and an amount Q_{hyb} of thermal energy at a single level of temperature T_Q^{hyb}, while consuming on a yearly basis an amount P_R^{hyb} of renewable primary energy and an amount P_F^{hyb} of fossil primary energy. Accordingly, we adopt symbols and subscripts W, Q, and P, respectively, for work (electricity), heat (thermal energy), and primary energy, while the subscripts R and F stand for renewable and fossil, respectively.

This can be the case of a hybrid natural-gas or coal cogeneration facility integrated with a biomass or solar energy input. For simplicity, it is assumed that the hybrid cogeneration facility is included in a local area where all the other heat production and power production facilities are single-resource and single-product, each producing either heat or electricity and using either a renewable or a fossil resource. A sketch of the local area is represented in Fig. 1 where the energy fluxes between input and output of each facility can be defined as follows. In case of the renewable-only electricity plant, renewable primary energy $P^{sr}_{\substack{R\\W}}$ is converted into electricity W_R^{sr} according to the plant primary energy factor $\bar{f}^{sr}_{\substack{R\\W}}$.

Similar considerations are valid for the other three single-resource

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[1] We recall that the “primary energy factor” of a given good is defined as the amount of primary energy that is consumed to produce a unit amount of that good, taking into consideration all processes in its life cycle. In the case of a power plant it equals the inverse of the conversion efficiency calculated on the basis of the overall primary energy consumption, i.e., not just the direct consumption in the facility itself but also in all the other processes in its life cycle. For example, for natural gas Ref. [20] suggests to increment the actual consumption in the power plant by 10% to account for the gas extraction and pipelining consumptions.

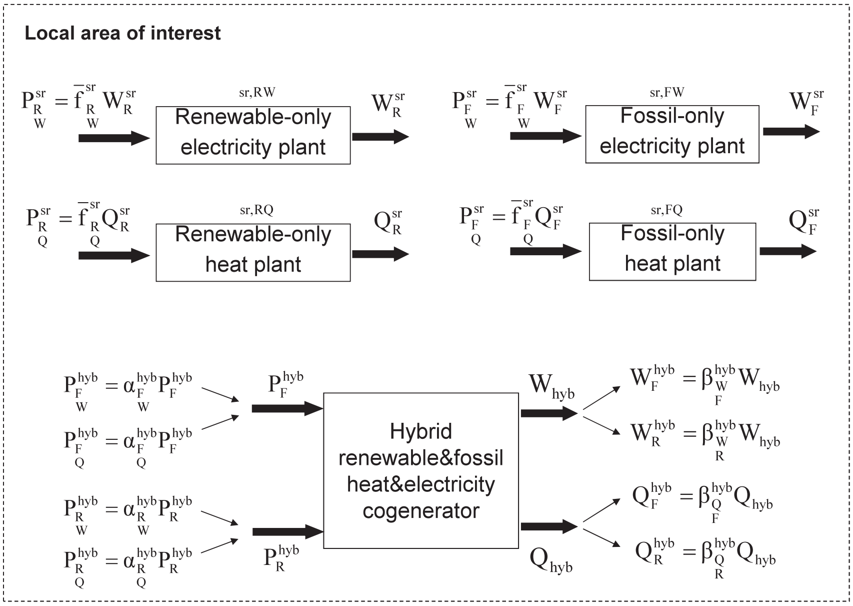

Fig. 1. Schematic representation of a local area of interest with a single hybrid cogeneration plant.

and single-product facilities shown in the upper part of Fig. 1. For the hybrid cogenerator sketched in the lower part of Fig. 1, the input primary energy sources P^{hyb}_F and P^{hyb}_R can be conceptually divided between the quota used to produce electricity and heat, namely, $P^{hyb}_F = P^{hyb}_{\substack{F\\W}} + P^{hyb}_{\substack{F\\Q}}$ and $P^{hyb}_R = P^{hyb}_{\substack{R\\W}} + P^{hyb}_{\substack{R\\Q}}$. In turn, the produced electricity and heat can be apportioned between the renewable and fossil quota, namely, $W_{hyb} = W^{hyb}_R + W^{hyb}_F$ and $Q_{hyb} = Q^{hyb}_R + Q^{hyb}_F$.

The objective of the analysis in the first part of this paper is to define methods to allocate the heat and power productions of the hybrid fossil-solar cogeneration facility among the different primary resources it consumes. This is equivalent to finding the eight unknowns of the allocation problem, i.e., the four resource allocation fraction α's and the four product allocation fractions β's defined as follows

$$\alpha^{hyb}_{\substack{R\\W}} = \frac{P^{hyb}_{\substack{R\\W}}}{P^{hyb}_R} \quad \alpha^{hyb}_{\substack{R\\Q}} = \frac{P^{hyb}_{\substack{R\\Q}}}{P^{hyb}_R} \quad \alpha^{hyb}_{\substack{F\\W}} = \frac{P^{hyb}_{\substack{F\\W}}}{P^{hyb}_F} \quad \alpha^{hyb}_{\substack{F\\Q}} = \frac{P^{hyb}_{\substack{F\\Q}}}{P^{hyb}_F}$$
$$\beta^{hyb}_{\substack{W\\R}} = \frac{W^{hyb}_R}{W_{hyb}} \quad \beta^{hyb}_{\substack{Q\\R}} = \frac{Q^{hyb}_R}{Q_{hyb}} \quad \beta^{hyb}_{\substack{W\\F}} = \frac{W^{hyb}_F}{W_{hyb}} \quad \beta^{hyb}_{\substack{Q\\F}} = \frac{Q^{hyb}_F}{Q_{hyb}} \tag{1}$$

Thus, $P^{hyb}_{\substack{R\\W}}$ and $P^{hyb}_{\substack{R\\Q}}$ are the fair shares of the consumption P^{hyb}_R of primary renewable energy in the hybrid cogeneration facility that can be attributed to the produced electricity and heat, respectively; $P^{hyb}_{\substack{F\\W}}$ and $P^{hyb}_{\substack{F\\Q}}$ the respective fair shares of the consumption P^{hyb}_F of primary fossil energy; W^{hyb}_R and W^{hyb}_F are the fair shares of the production W_{hyb} of electricity that can be attributed to the consumption of renewable or fossil resource, respectively; Q^{hyb}_R and Q^{hyb}_F the respective shares of the production Q_{hyb} of heat. Moreover, we clearly have the conditions

$$\alpha^{hyb}_{\substack{R\\W}} + \alpha^{hyb}_{\substack{R\\Q}} = 1 \quad \alpha^{hyb}_{\substack{F\\W}} + \alpha^{hyb}_{\substack{F\\Q}} = 1 \quad \beta^{hyb}_{\substack{W\\R}} + \beta^{hyb}_{\substack{W\\F}} = 1 \quad \beta^{hyb}_{\substack{Q\\R}} + \beta^{hyb}_{\substack{Q\\F}} = 1 \tag{2}$$

For example, the renewable-resource allocation fraction $\alpha^{hyb}_{\substack{R\\W}}$ represents the fraction of the overall consumption of the renewable resource which is to be considered as used to produce electricity, $\alpha^{hyb}_{\substack{R\\Q}} = 1 - \alpha^{hyb}_{\substack{R\\W}}$ to produce heat. Again, the electricity allocation fraction $\beta^{hyb}_{\substack{W\\R}}$ represents the fraction of the overall electricity production that is to be considered as obtained from the renewable resource, $\beta^{hyb}_{\substack{W\\F}} = 1 - \beta^{hyb}_{\substack{W\\R}}$ from the fossil resource.

The rationale of all the allocation methods considered here, is that the allocation compares the hybrid cogenerator with a particular reference energy production scenario, characterized by reference partial conversion efficiencies (or primary energy factors) for each resource–product pair (in our case fossil-electricity, fossil-heat, renewable-electricity, renewable-heat). In Section 6, we discuss some of the possible purposes of allocation, such as to measure primary energy savings, to compare partial efficiencies and environmental impact, to design energy policies, to distribute incentives, etc., and we suggest that each different purpose may require a different reference energy production scenario, to be identified by the regulator or some local authority.

To allocate the co-consumed fossil and renewable resources among the heat Q_{hyb} and electricity W_{hyb} produced by the hybrid-cogeneration plant, the general rule is that the two resources are allocated in proportion to the corresponding consumptions of fossil and renewable resources, respectively, that would be required in the assumed reference scenario to produce the same yearly amounts of heat, Q_{hyb}, and electricity, W_{hyb}. In terms of equations this is expressed by

$$\alpha^{hyb}_{\substack{F\\W}} = \frac{f^{x}_{\substack{F\\W}} W_{hyb}}{f^{x}_{\substack{F\\W}} W_{hyb} + f^{x}_{\substack{F\\Q}} Q_{hyb}} \qquad \alpha^{hyb}_{\substack{F\\Q}} = \frac{f^{x}_{\substack{F\\Q}} Q_{hyb}}{f^{x}_{\substack{F\\W}} W_{hyb} + f^{x}_{\substack{F\\Q}} Q_{hyb}}$$
$$\alpha^{hyb}_{\substack{R\\W}} = \frac{f^{x}_{\substack{R\\W}} W_{hyb}}{f^{x}_{\substack{R\\W}} W_{hyb} + f^{x}_{\substack{R\\Q}} Q_{hyb}} \qquad \alpha^{hyb}_{\substack{R\\W}} = \frac{f^{x}_{\substack{R\\W}} W_{hyb}}{f^{x}_{\substack{R\\W}} W_{hyb} + f^{x}_{\substack{R\\Q}} Q_{hyb}} \tag{3}$$

where the terms $f^{x}_{\substack{F\\W}}$, $f^{x}_{\substack{F\\Q}}$, $f^{x}_{\substack{R\\W}}$ and $f^{x}_{\substack{R\\Q}}$ are the reference partial primary energy factors that characterize the assumed reference scenario and hence the resource allocation method. More precisely, we may call $f^{x}_{\substack{R\\W}}$ and $f^{x}_{\substack{R\\Q}}$ the reference "renewable partial primary energy factors" of electricity and heat, and $f^{x}_{\substack{F\\W}}$ and $f^{x}_{\substack{F\\Q}}$ the reference "fossil partial primary energy factors" of electricity and heat. Clearly, the terms $f^{x}_{\substack{F\\W}} W_{hyb}$ and $f^{x}_{\substack{R\\W}} W_{hyb}$ represent respectively the consumptions of fossil and renewable resources required in the assumed reference scenario to produce the electricity W_{hyb}. Similarly, $f^{x}_{\substack{F\\Q}} Q_{hyb}$ and $f^{x}_{\substack{R\\Q}} Q_{hyb}$ are the consumptions of fossil and renewable resources required in the reference scenario to produce the heat Q_{hyb}.

Similarly, to allocate the cogenerated electricity and heat among the fossil resource P^{hyb}_{F} and the renewable resource P^{hyb}_{R} consumed by the hybrid-cogeneration plant, the general rule is that the two-products are allocated in proportion to the corresponding productions of electricity and heat, respectively, that would be obtained in the assumed reference scenario from the consumption of the same yearly amounts of fossil resource, P^{hyb}_{F}, and renewable resource, P^{hyb}_{R}. In terms of equations this is expressed by

$$\beta^{hyb}_{\substack{W\\R}} = \frac{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}}}}{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}}} + \dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}}}} \qquad \beta^{hyb}_{\substack{W\\F}} = \frac{\dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}}}}{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}}} + \dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}}}}$$
$$\beta^{hyb}_{\substack{Q\\R}} = \frac{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\Q}}}}{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\Q}}} + \dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\Q}}}} \qquad \beta^{hyb}_{\substack{Q\\F}} = \frac{\dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\Q}}}}{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\Q}}} + \dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\Q}}}} \tag{4}$$

Clearly, the terms $P^{hyb}_{F}/f^{x}_{\substack{F\\W}}$ and $P^{hyb}_{R}/f^{x}_{\substack{R\\W}}$ represent the productions of electricity that are obtained in the assumed reference scenario by consuming P^{hyb}_{F} of fossil primary energy and P^{hyb}_{R} of renewable primary energy, respectively. Similarly, $P^{hyb}_{F}/f^{x}_{\substack{F\\Q}}$ and $P^{hyb}_{R}/f^{x}_{\substack{R\\Q}}$ represent the productions of heat that are obtained in the assumed reference scenario by consuming P^{hyb}_{F} of fossil primary energy and P^{hyb}_{R} of renewable primary energy, respectively.

As anticipated in the introduction, it can be observed that we are extending the resources allocation approaches previously discussed in Ref. [13] for combined heat and power production, and the products allocation approaches for hybrid fossil-solar power plants addressed in Ref. [15]. In fact, the resource allocation fractions α's defined by Eq. (3) are the equivalent to the two allocation fractions defined by Eq. (2) in Ref. [13] for the case of a single-resource allocation. Clearly in the case considered here, we have four of such parameters, due to the allocation of both the fossil and solar primary resources. In the same fashion the four product allocation fractions β's defined by Eq. (4) correspond to the two allocation fractions obtained in Eq. (10) of Ref. [15] for case of single-product allocation.

Once the reference partial primary energy factors $f^{x}_{\substack{F\\W}}$, $f^{x}_{\substack{F\\Q}}$, $f^{x}_{\substack{R\\W}}$, $f^{x}_{\substack{R\\Q}}$ are specified, the system of Eqs. (3) and (4) determines the eight allocation fractions α's and β's, which represent the solution of the allocation problem.

Once the allocation fractions are found, they may be used to compute the partial primary energy factors of the hybrid cogenerator. Indeed, the meaning of the allocation is that of the electrical energy W_{hyb} produced by the hybrid cogeneration plant, the fraction $W^{hyb}_{F} = \beta^{hyb}_{\substack{W\\F}} W_{hyb}$ is obtained from the fossil fuel while the fraction $W^{hyb}_{R} = \beta^{hyb}_{\substack{W\\R}} W_{hyb}$ is obtained from the renewable resource (this amount should qualify for subsidies reserved to renewable-to-electricity conversion). Similarly, of the heat Q_{hyb} produced, $Q^{hyb}_{F} = \beta^{hyb}_{\substack{Q\\F}} Q_{hyb}$ is obtained from the fossil fuel and $Q^{hyb}_{R} = \beta^{hyb}_{\substack{Q\\R}} Q_{hyb}$ from the renewable resource (this amount should qualify for subsidies reserved to renewable-to-heating conversion). Conversely, of the fossil-fuel primary energy consumption P^{hyb}_{F} in the facility, $P^{hyb}_{\substack{F\\W}} = \alpha^{hyb}_{\substack{F\\W}} P^{hyb}_{F}$ is used to produce electricity and $P^{hyb}_{\substack{F\\Q}} = \alpha^{hyb}_{\substack{F\\Q}} P^{hyb}_{F}$ to produce heat ($P^{hyb}_{\substack{F\\W}}$ and $P^{hyb}_{\substack{F\\Q}}$ should be used to determine the greenhouse gas fingerprints of the produced electricity and heat, respectively). Similarly, of the renewable-resource primary energy consumption P^{hyb}_{R} in the facility, $P^{hyb}_{\substack{R\\W}} = \alpha^{hyb}_{\substack{R\\W}} P^{hyb}_{R}$ is used to produce electricity and $P^{hyb}_{\substack{R\\Q}} = \alpha^{hyb}_{\substack{R\\Q}} P^{hyb}_{R}$ to produce heat.

Therefore, for each resource–product pair, the partial primary energy factors for the hybrid plant can be defined as follows

$$f^{hyb}_{\substack{R\\W}} = \frac{\alpha^{hyb}_{\substack{R\\W}} P^{hyb}_{R}}{\beta^{hyb}_{\substack{W\\R}} W_{hyb}} \qquad f^{hyb}_{\substack{F\\W}} = \frac{\alpha^{hyb}_{\substack{F\\W}} P^{hyb}_{F}}{\beta^{hyb}_{\substack{W\\F}} W_{hyb}}$$
$$f^{hyb}_{\substack{R\\Q}} = \frac{\alpha^{hyb}_{\substack{R\\Q}} P^{hyb}_{R}}{\beta^{hyb}_{\substack{Q\\R}} Q_{hyb}} \qquad f^{hyb}_{\substack{F\\Q}} = \frac{\alpha^{hyb}_{\substack{F\\Q}} P^{hyb}_{F}}{\beta^{hyb}_{\substack{Q\\F}} Q_{hyb}} \tag{5}$$

It is important to make clear that the partial primary energy factors $f^{hyb}_{\substack{R\\W}}$, $f^{hyb}_{\substack{R\\Q}}$, $f^{hyb}_{\substack{F\\W}}$, and $f^{hyb}_{\substack{F\\Q}}$ represent, respectively, the portions of the overall electricity or heat produced which we allocate to either the renewable or the fossil resource, and they must be distinguished from the primary energy factors of the renewable and fossil resources used by the hybrid cogeneration facility,

$$f^{hyb}_{R} = \frac{P^{hyb}_{R}}{E^{hyb}_{R}} \text{ and } f^{hyb}_{F} = \frac{P^{hyb}_{F}}{E^{hyb}_{F}} \tag{6}$$

as well as from the average primary energy factor of the mix of resources it uses

$$f_{hyb} = \frac{P^{hyb}_{F} + P^{hyb}_{R}}{E^{hyb}_{F} + E^{hyb}_{R}} \tag{7}$$

where in Eqs. (6) and (7) E^{hyb}_{F} and E^{hyb}_{R} are the fuel energy (based on lower heating value) of the fossil and renewable resources, respectively.

Finally and importantly, it is now possible to define and compute as follows the partial energy-conversion efficiencies and COP (coefficients of performance) of the hybrid cogenerator. The efficiency of conversion of fossil-fuel energy to electricity is

$$\eta^{hyb}_{F_W} = \frac{\beta^{hyb}_{W_F} W_{hyb}}{\alpha^{hyb}_{F_W} P^{hyb}_{F} / f^{hyb}_{F}} = \frac{f^{hyb}_{F}}{f^{hyb}_{F_W}} \tag{8}$$

The coefficient of performance of conversion of fossil-fuel energy to heat is

$$COP^{hyb}_{F_Q} = \frac{\beta^{hyb}_{Q_F} Q_{hyb}}{\alpha^{hyb}_{F_Q} P^{hyb}_{F} / f^{hyb}_{F}} = \frac{f^{hyb}_{F}}{f^{hyb}_{F_Q}} \tag{9}$$

The efficiency of conversion of renewable-resource energy to electricity is

$$\eta^{hyb}_{R_W} = \frac{\beta^{hyb}_{W_R} W_{hyb}}{\alpha^{hyb}_{R_W} P^{hyb}_{R} / f^{hyb}_{R}} = \frac{f^{hyb}_{R}}{f^{hyb}_{R_W}} \tag{10}$$

The coefficient of performance of conversion of renewable-resource energy to heat is

$$COP^{hyb}_{R_Q} = \frac{\beta^{hyb}_{Q_R} Q_{hyb}}{\alpha^{hyb}_{R_Q} P^{hyb}_{R} / f^{hyb}_{R}} = \frac{f^{hyb}_{R}}{f^{hyb}_{R_Q}} \tag{11}$$

If the method adopted for allocation is “fair”, then the above efficiencies and COP's represent a fair apportionment of the benefits of hybridization among the different resources and of cogeneration among the different products, thus allowing fair terms of comparison between the efficiencies and COP's of the hybrid cogenerator and the respective single-resource single-production efficiencies and COP chosen by some local authority or regulator as threshold values to access subsidies or to assess the sustainability with respect to single-resource single-production facilities in a local area, characterized by $\eta^{sr}_{F_W}$, $COP^{sr}_{F_Q}$, $\eta^{sr}_{R_W}$, $COP^{sr}_{R_Q}$.

In Section 4 we turn to examining how the reference partial primary energy factors $f^{x}_{F_W}$, $f^{x}_{F_Q}$, $f^{x}_{R_W}$, and $f^{x}_{R_Q}$ are specified in the different allocation methods considered in the present paper and compare for the case study defined in Section 3.

3. Case study

To provide a quantitative analysis and comparison of the different allocation approaches in the case of a hybrid cogeneration facility, let us refer to a local area of interest as sketched in Fig. 1, that we consider as representative of a generic mix of industrial, residential, and tertiary activities with yearly thermal consumption twice the consumption of electricity.

We assume that initially the annual electricity and heat demand are supplied according to the following shares:

- 90% of the electricity by fossil-only (i.e., natural-gas single-resource) power plants operating with $f^{sr}_{F_W} = f^{sr,FW}_{F}/\eta^{sr}_{FW}$, assuming an overall (yearly) average efficiency $\eta^{sr}_{FW} = 0.38$ and a fuel primary energy factor $f^{sr,FW}_{F} = 1.1$ (natural gas); the remaining 10% of electricity by renewable-only (i.e., single-resource, non-hybrid solar) power plants operating with $f^{sr}_{R_W} = f^{sr,RW}_{R}/\eta^{sr}_{RW}$, assuming an overall (yearly) average efficiency $\eta^{sr}_{RW} = 0.153$ and a primary energy factor $f^{sr,RW}_{R} = 1$;
- 90% of the heat by fossil-only (i.e., natural-gas single-resource) heat plants operating with $f^{sr}_{F_Q} = f^{sr,FQ}_{F}/COP^{sr}_{F_Q}$, assuming an overall (yearly) average efficiency $COP^{sr}_{F_Q} = 0.9$ and a primary energy factor $f^{sr,FQ}_{F} = 1.1$ (natural gas); the remaining 10% of heat by renewable-only (i.e., solar single-resource) heat plants operating with $f^{sr}_{R_Q} = f^{sr,RQ}_{R}/COP^{sr}_{R_Q}$, assuming an overall (yearly) average efficiency $COP^{sr}_{R_Q} = 0.5$ and a primary energy factor $f^{sr,RQ}_{R} = 1.0$.

Then we consider that the fossil-only facilities (both electricity and heat plants) are progressively replaced by hybrid renewable-fossil heat-electricity cogenerators (Fig. 1) based on the same SICCS (Solar-Integrated Combined-Cycle System) technology with parabolic trough solar field considered in the case study proposed in Ref. [15]. However, since in the present case study the facility is operated in cogeneration mode, it is assumed that a fraction of the steam is extracted from the low pressure section of the steam turbine. This solution, which is common practice in the operation of cogeneration combined cycles, allows a production of heat with a limited reduction in the production of electricity. To compute the annual energy balance of the cogeneration SICCS, which is a necessary input of every allocation method, the following additional hypotheses are made:

- the input primary energies are the same as in the case of the non-cogeneration SICCS [15], i.e. $P^{hyb}_{F} = 1878$ GWh and $P^{hyb}_{R} = 425$ GWh;
- the reference value of 867 GWh of yearly production of electricity when the plant is operated in non-cogeneration mode, is reduced by 10% in cogeneration mode, resulting in $W_{hyb} = 780$ GWh;
- the thermal energy to reduced electrical production ratio is assumed equal to 5, i.e., the cogenerative mode yields a 1 kWh

Table 1
Summary of assumptions made and values of some important parameters for the hybrid solar & fossil/heat & electricity cogeneration case study.

Parameters of the local area	
Yearly electricity to heat demand ratio, $W^{loc}_{tot}/Q^{loc}_{tot}$	0.5
Renewable-only share of power production in the local area, $\gamma^{sr,RW}_{W}$	0.1
Renewable-only share of heat production in the local area, $\gamma^{sr,RQ}_{Q}$	0.1
Primary energy factor of solar energy for solar-only heat facilities in the local area, $f^{sr,RW}_{R}$, $f^{sr,RQ}_{R}$	1.0
Average efficiency of the renewable-only electricity facilities, $\eta^{sr}_{\substack{R\\W}}$	0.153
Average efficiency of the renewable-only heat facilities, $COP^{sr}_{\substack{R\\Q}}$	0.5
Primary energy factor of natural gas (assumed the only fuel used in the local area for fossil-only facilities), $f^{sr,FW}_{F}$, $f^{sr,FQ}_{F}$	1.1
Average efficiency of the fossil-only electricity facilities, $\eta^{sr}_{\substack{F\\W}}$	0.38 (0.55)
Average efficiency of the fossil-only heat facilities, $COP^{sr}_{\substack{F\\Q}}$	0.90
Parameters of the hybrid-cogeneration facilities	
Electric index W^{hyb}/Q^{hyb}	780/434
Fossil to renewable primary energy ratio, $\sigma_{hyb} = P^{hyb}_{F}/P^{hyb}_{R}$	1878/425
Fossil primary energy to electricity ratio, P^{hyb}_{F}/W_{hyb}	1878/780
Renewable primary energy to electricity ratio, P^{hyb}_{R}/W_{hyb}	425/780
Fossil primary energy to heat ratio, P^{hyb}_{F}/Q_{hyb}	1878/434
Renewable primary energy to heat ratio, P^{hyb}_{R}/Q_{hyb}	425/434

reduction in the production of electricity with respect to the non-cogenerative mode for every 5 kWh of cogenerated heat, resulting in $Q_{hyb} = 434$ GWh.

We further assume that the installation of hybrid cogenerators proceeds in the local area until all the single-resource fossil power plants are replaced, i.e., when W^{sr}_{F} is eventually entirely replaced by W_{hyb}. It is worth noting that this situation represents the condition of highest possible degree of penetration of cogeneration for the considered local area of interest.[2]

Finally, regarding the renewable single-resource facilities, it is assumed that both W^{sr}_{R} and Q^{sr}_{R} remain fixed to their initial value. Table 1 summarizes the assumptions considered in the case study.

4. Allocation methods based on different choices of the prescribed reference efficiencies

4.1. SRSPR (Single-Resource Separate-Production Reference allocation method)

This allocation method assumes the following rule

$$f^{x}_{\substack{F\\W}} = f^{ref}_{\substack{F\\W}} \quad f^{x}_{\substack{F\\Q}} = f^{ref}_{\substack{F\\Q}} \quad f^{x}_{\substack{R\\W}} = f^{ref}_{\substack{R\\W}} \quad f^{x}_{\substack{R\\Q}} = f^{ref}_{\substack{R\\Q}} \tag{12}$$

where $f^{ref}_{\substack{R\\W}}$, $f^{ref}_{\substack{F\\W}}$, $f^{ref}_{\substack{R\\Q}}$, and $f^{ref}_{\substack{F\\Q}}$ are reference partial primary energy factors chosen by some authority as representative of the performance of the (best available or representative average, usually single-resource) power and heat production technologies that use, respectively, the same renewable resource and the same fossil fuel as the hybrid cogeneration facility.

The solution of the allocation problem is obtained by substituting Eq. (12) into Eqs. (3) and (4) and solving for the resource and product allocation fractions α's and β's. Table 2 lists the resulting values of the various parameters for the case study defined in Section 3, for both the case of $\eta^{ref}_{\substack{F\\W}} = 0.38$ and 0.55. It is noteworthy that the benefits of hybridization and cogeneration are significant with respect to all single-resource single-production (SRSP) facilities in the case of $\eta^{ref}_{\substack{F\\W}} = 0.38$, and for all except for $COP^{hyb}_{\substack{F\\Q}}$ and $\eta^{hyb}_{\substack{R\\W}}$ for the case of $\eta^{ref}_{\substack{F\\W}} = 0.55$. In particular,

- the efficiency of conversion of fossil-fuel energy to electricity goes from the 38% (55%) of the SRSP facility to the 51.28% (57.26%) of the hybrid facility,
- the coefficient of performance of conversion of fossil-fuel energy to heat goes from the 90% of the SRSP facility to the 117.4% (88.01%) of the hybrid facility,
- the efficiency of conversion of renewable-resource energy to electricity goes from the 15.3% of the SRSP facility to the 19.57% (13.91%) of the hybrid facility,

Table 2
Summary of SRSPR and ExRR allocation results for the hybrid solar & fossil/heat & electricity cogeneration case study defined in Section 3 (parameters summarized in Table 1).

	SRSPR		ExRR	
	$\eta^{sr}_{\substack{F\\W}} = 0.38$	$\eta^{sr}_{\substack{F\\W}} = 0.55$	$T^{hyb}_{Q} = 95\ ^{\circ}C$	$T^{hyb}_{Q} = 150\ ^{\circ}C$
$\alpha^{hyb}_{\substack{R\\W}}$	0.8545	0.8545	0.9043	0.8588
$\alpha^{hyb}_{\substack{R\\Q}}$	0.1455	0.1455	0.0957	0.1412
$\alpha^{hyb}_{\substack{F\\W}}$	0.8098	0.7463	0.9043	0.8588
$\alpha^{hyb}_{\substack{F\\Q}}$	0.1902	0.2537	0.0957	0.1412
$\beta^{hyb}_{\substack{W\\R}}$	0.0911	0.0648	0.1739	0.1739
$\beta^{hyb}_{\substack{W\\F}}$	0.9089	0.9352	0.8261	0.8261
$\beta^{hyb}_{\substack{Q\\R}}$	0.1215	0.1215	0.1739	0.1739
$\beta^{hyb}_{\substack{Q\\F}}$	0.8785	0.8785	0.8261	0.8261
$f^{hyb}_{\substack{R\\W}}$	5.1109	7.1891	2.8339	2.6913
$f^{hyb}_{\substack{F\\W}}$	2.1451	1.9212	2.6355	2.5029
$f^{hyb}_{\substack{R\\Q}}$	1.1727	1.1727	0.5390	0.7953
$f^{hyb}_{\substack{F\\Q}}$	0.9370	1.2499	0.5013	0.7396
χ^{hyb}_{W}	0.4429	0.3060	0.9300	0.9300
χ^{hyb}_{Q}	0.6111	0.6111	0.9300	0.9300
Φ^{hyb}_{F}	0.4222	0.6111	0.1902	0.2955
Φ^{hyb}_{R}	0.3060	0.3060	0.1902	0.2955
$\eta^{hyb}_{\substack{F\\W}}$	0.5128	0.5726	0.4174	0.4395
$COP^{hyb}_{\substack{F\\Q}}$	1.1739	0.8801	2.1942	1.4872
$\eta^{hyb}_{\substack{R\\W}}$	0.1957	0.1391	0.3529	0.3716
$COP^{hyb}_{\substack{R\\Q}}$	0.8528	0.8528	1.8551	1.2574

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[2] This case occurs whenever the initial value of the ratio W^{sr}_{F}/Q^{sr}_{F} is lower than the electric index W_{hyb}/Q_{hyb} of the hybrid cogeneration plant (see Ref. [13]).

- the coefficient of performance of conversion of renewable-resource energy to heat goes from the 50% of the SRSP facility to the 85.28% of the hybrid facility.

4.2. ExRR (Exergy-based Reversible-Reference allocation method)

According to this method we assume

$$f^{x}_{\substack{F\\W}} = f^{Ex}_{\substack{F\\W}} \quad f^{x}_{\substack{F\\Q}} = f^{Ex}_{\substack{F\\Q}} \quad f^{x}_{\substack{R\\W}} = f^{Ex}_{\substack{R\\W}} \quad f^{x}_{\substack{R\\Q}} = f^{Ex}_{\substack{R\\Q}} \tag{13}$$

where

$$\begin{aligned}
f^{Ex}_{\substack{F\\W}} &= \frac{P^{hyb}_{F}}{W^{hyb}_{F,ideal}} = \frac{P^{hyb}_{F}}{Ex^{hyb}_{F}}\\
f^{Ex}_{\substack{R\\W}} &= \frac{P^{hyb}_{R}}{W^{hyb}_{R,ideal}} = \frac{P^{hyb}_{R}}{Ex^{hyb}_{R}}\\
f^{Ex}_{\substack{F\\Q}} &= \frac{P^{hyb}_{F}}{Q^{hyb}_{F,ideal}} = \frac{P^{hyb}_{F}}{Ex^{hyb}_{F}}\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)\\
f^{Ex}_{\substack{R\\Q}} &= \frac{P^{hyb}_{R}}{Q^{hyb}_{R,ideal}} = \frac{P^{hyb}_{R}}{Ex^{hyb}_{R}}\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)
\end{aligned} \tag{14}$$

where T_{env} is the temperature of the environment, $T^{hyb}_{Q} = (h_{feed} - h_{return})/(s_{feed} - s_{return})$ is the exergy-equivalent-single-heat-source delivery temperature, with h_{feed}, s_{feed} and h_{return}, s_{return} respectively the enthalpy and entropy of the feed and return streams with which the cogeneration facility delivers the thermal energy. Clearly, for heat delivered in the form of heating a gas or liquid stream with negligible pressure drop and constant specific heat capacity, T^{hyb}_{Q} is the log-mean temperature $T^{hyb}_{Q} = (T_{feed} - T_{return})/\ln(T_{feed}/T_{return})$. In writing Eq. (14) we take into account that the electricity productions by reversible processes, $W^{hyb}_{F,ideal}$ and $W^{hyb}_{R,ideal}$, equal the respective exergies of the consumed resources, Ex^{hyb}_{F} and Ex^{hyb}_{R}, while the heat productions by reversible processes, $Q^{hyb}_{F,ideal}$ and $Q^{hyb}_{R,ideal}$, are related to the exergies of the consumed resources by $Q^{hyb}_{F,ideal} = Ex^{hyb}_{F}/\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)$ and $Q^{hyb}_{R,ideal} = Ex^{hyb}_{R}/\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)$, respectively.

Substitution of Eq. (14) into Eqs. (3) and (4) yields

$$\begin{aligned}
\alpha^{hyb}_{\substack{R\\W}} = \alpha^{hyb}_{\substack{F\\W}} &= \frac{W_{hyb}}{W_{hyb} + Q_{hyb}\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)}\\
\alpha^{hyb}_{\substack{R\\Q}} = \alpha^{hyb}_{\substack{F\\Q}} &= \frac{Q_{hyb}\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)}{W_{hyb} + Q_{hyb}\left(1-\frac{T_{env}}{T^{hyb}_{Q}}\right)}\\
\beta^{hyb}_{\substack{R\\W}} = \beta^{hyb}_{\substack{R\\Q}} &= \frac{Ex^{hyb}_{R}}{Ex^{hyb}_{R} + Ex^{hyb}_{F}}\\
\beta^{hyb}_{\substack{F\\W}} = \beta^{hyb}_{\substack{F\\Q}} &= \frac{Ex^{hyb}_{F}}{Ex^{hyb}_{R} + Ex^{hyb}_{F}}
\end{aligned} \tag{15}$$

It is clear that the exergy-based method is equivalent to an SRSPR method in which the authority sets as reference efficiencies (for the energy conversions of fossil/renewable primary energies into electricity/heat) those of thermodynamically reversible machinery. In other words, we can say that the exergy method is a Reversible-Reference version of the SRSPR method, hence the acronym ExRR. As such, this method will become fairer as technological advances will get us closer to thermodynamic reversibility. But it is unfair with current technologies because, as already noted in Ref. [13], current power production technologies have average second-law efficiencies much closer to 100% than current heat production technologies. Thus, when compared with the SRSPR method, the ExRR method credits the thermal energy with too high a share of the cogeneration benefits leaving an unfairly little share of the fuel savings to the cogenerated electricity. Moreover, as already noted in Ref. [15], it is unfair also because current fossil fuel technologies have average second-law efficiencies much closer to 100% than current renewable technologies. Thus, when compared with the SRSPR method, the ExRR method credits the renewable resources with too high a share of the hybridization benefits leaving an unfairly little share of the enhanced production to the fossil fuel consumption.

In other words, though based on sound thermodynamic reasoning, the exergy-based allocation method assumes as references, hypothetical efficiencies that are too distant from the average efficiencies of the current industrial and technological scenario. Therefore, if adopted as a basis of regulations, this method would result in market distortions which may for example give too much advantage to hybrid district heating systems thus improperly discouraging home owners that have access to such heating systems from investing in energy-saving improvements such as better building and window insulation as well as individual uses of solar energy.

Table 2 lists the resulting values of the various parameters for the case study defined in Section 3, assuming $T^{hyb}_{Q} = 368$ K (=95°C, typical yearly average value for a district heating system) and $T^{hyb}_{Q} = 423$ K (=150°C suitable for some industrial processes). Fossil and renewable exergies are computed respectively as[3] $Ex^{hyb}_{F} = P^{hyb}_{F} = 1878$ GWh and $Ex^{hyb}_{R} = 0.93\,P^{hyb}_{R} = 395$ GWh. This choice is based on the fact that, on one hand, for most hydrocarbons $Ex^{hyb}_{F} \approx E^{hyb}_{F}$ to within ±2.5% (see, e.g. Ref. [21]) and, on the other hand, for solar radiation it is typical to assume $Ex^{hyb}_{R} \approx 0.93\,E^{hyb}_{R}$ (see, e.g. Refs. [22,23]).

Results reported in Table 2 suggest the following comments:

- The fraction of primary resources allocated to the heat is higher for $T^{hyb}_{Q} = 150°\mathrm{C}$ ($\alpha^{hyb}_{\substack{F\\Q}} = \alpha^{hyb}_{\substack{R\\Q}} = 0.141$) than for $T^{hyb}_{Q} = 95°\mathrm{C}$ ($\alpha^{hyb}_{\substack{F\\Q}} = \alpha^{hyb}_{\substack{R\\Q}} = 0.096$) due to the higher exergy associated to the higher temperature.
- Conversely, the value of T^{hyb}_{Q} has no effect on the product allocation fractions β's, as they depend only on the exergy of the fuels, according to third and fourth of Eq. (15).
- Comparing the values of $COP^{hyb}_{\substack{F\\Q}}$ and $COP^{hyb}_{\substack{R\\Q}}$ with those of the SRSPR method, it is confirmed that the ExRR method provides

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[3] The question of how much exergy should be associated with incident solar radiation on the Earth surface is not yet fully resolved and we refer to the extensive discussions available in the literature, in particular see Refs. [21,22] and references therein.

too much credit to the thermal energy with respect to the state of the art technology of heat production. Also the unfairly high values of $\eta^{\text{hyb}}_{\substack{R\\W}}$ are a consequence of the lower second-law efficiencies of the current renewable technologies with respect to the fossil conversion, as already observed in the discussion above.

5. STALPR allocation method

As reported in Refs. [13 and 15], one limitation of the classical allocation criteria is that they are based on some fixed prescribed reference efficiencies of conversion for each resource-to-product pair. These reference efficiencies are to be assigned by some authority and in general differ from the actual average ones that characterize the energy portfolio of the local area in which the facility under consideration is located or with which it is to be compared. Being fixed, the reference values are not dynamically influenced by the installation of new facilities nor by the progressive penetration of more efficient technologies within the local area. Thus, effects due to the modification of the local energy portfolio are neglected. This fact may result in distortions of the local energy market, unless the authority continuously updates the reference efficiencies by keeping into constant account the progressive penetration of different facilities within the local area.

Our reasoning presumes that our definition of “fair allocation” is as follows: it must be based on reference efficiencies that are representative of the actual average efficiencies of the energy production portfolio with which the resulting efficiencies of the facility are to be compared. In our illustrative examples, to fix ideas we assume that such portfolio is that of the local area where the hybrid cogenerator is located. But other relevant options, i.e., other prescribed comparison contexts, are possible and may characterize particular objectives of an energy policy. For instance, to promote and maintain a healthy competition towards continuous efficiency improvements, it may be useful to take as reference portfolio the (dynamically changing) subset of existing facilities in a given region or nation that adopt the same technology of the hybrid cogenerator.

To overcome the drawbacks arising from fixed reference efficiencies, we propose a self-consistent method whereby the allocation is adaptive and self-tuned to the local energy production portfolio. We call it the STALPR (Self-Tuned-Average-Local-Productions-Reference) method. The method was first applied to the allocation of fuel primary energy [13] in cogeneration plants and then employed to define the renewable quota of the electricity produced in hybrid fossil-solar power plants [15]. In this section, the STALPR method is extended to the case of a hybrid fossil-solar heat-power cogenerator operating in a local area where all other electricity and heating facilities are non-cogeneration non-hybrid units.

The closure of the problem according to the STALPR method is given by following rules

$$f^{x}_{\substack{F\\W}} = f^{\text{loc}}_{\substack{F\\W}} \quad f^{x}_{\substack{F\\Q}} = f^{\text{loc}}_{\substack{F\\Q}} \quad f^{x}_{\substack{R\\W}} = f^{\text{loc}}_{\substack{R\\W}} \quad f^{x}_{\substack{R\\Q}} = f^{\text{loc}}_{\substack{R\\Q}} \tag{16}$$

where we have introduced the local partial primary energy factors, i.e., the average local renewable or fossil primary energy factors of heat and electricity, defined as follows

$$\begin{aligned} f^{\text{loc}}_{\substack{R\\W}} &= \frac{P^{\text{sr}}_{\substack{R\\W}} + P^{\text{hyb}}_{\substack{R\\W}}}{W^{\text{sr}}_{R} + W^{\text{hyb}}_{R}} \quad & f^{\text{loc}}_{\substack{F\\W}} &= \frac{P^{\text{sr}}_{\substack{F\\W}} + P^{\text{hyb}}_{\substack{F\\W}}}{W^{\text{sr}}_{F} + W^{\text{hyb}}_{F}} \\ f^{\text{loc}}_{\substack{R\\Q}} &= \frac{P^{\text{sr}}_{\substack{R\\Q}} + P^{\text{hyb}}_{\substack{R\\Q}}}{Q^{\text{sr}}_{R} + Q^{\text{hyb}}_{R}} \quad & f^{\text{loc}}_{\substack{F\\Q}} &= \frac{P^{\text{sr}}_{\substack{F\\Q}} + P^{\text{hyb}}_{\substack{F\\Q}}}{Q^{\text{sr}}_{F} + Q^{\text{hyb}}_{F}} \end{aligned} \tag{17}$$

Using Eq. (1), these may conveniently be rewritten in terms of the allocation fractions α's and β's as follows

$$\begin{aligned} f^{\text{loc}}_{\substack{R\\W}} &= \frac{P^{\text{sr}}_{\substack{R\\W}} + \alpha^{\text{hyb}}_{\substack{R\\W}} P^{\text{hyb}}_{R}}{W^{\text{sr}}_{R} + \beta^{\text{hyb}}_{\substack{W\\R}} W_{\text{hyb}}} \quad & f^{\text{loc}}_{\substack{F\\W}} &= \frac{P^{\text{sr}}_{\substack{F\\W}} + \alpha^{\text{hyb}}_{\substack{F\\W}} P^{\text{hyb}}_{F}}{W^{\text{sr}}_{F} + \beta^{\text{hyb}}_{\substack{W\\F}} W_{\text{hyb}}} \\ f^{\text{loc}}_{\substack{R\\Q}} &= \frac{P^{\text{sr}}_{\substack{R\\Q}} + \alpha^{\text{hyb}}_{\substack{R\\Q}} P^{\text{hyb}}_{R}}{Q^{\text{sr}}_{R} + \beta^{\text{hyb}}_{\substack{Q\\R}} Q_{\text{hyb}}} \quad & f^{\text{loc}}_{\substack{F\\Q}} &= \frac{P^{\text{sr}}_{\substack{F\\Q}} + \alpha^{\text{hyb}}_{\substack{F\\Q}} P^{\text{hyb}}_{F}}{Q^{\text{sr}}_{F} + \beta^{\text{hyb}}_{\substack{Q\\F}} Q_{\text{hyb}}} \end{aligned} \tag{18}$$

so that again, by substituting in Eqs. (3) and (4), we can solve for the eight unknowns $\alpha^{\text{hyb}}_{\substack{R\\W}}, \alpha^{\text{hyb}}_{\substack{R\\Q}}, \alpha^{\text{hyb}}_{\substack{F\\W}}, \alpha^{\text{hyb}}_{\substack{F\\Q}}, \beta^{\text{hyb}}_{\substack{W\\R}}, \beta^{\text{hyb}}_{\substack{Q\\R}}, \beta^{\text{hyb}}_{\substack{W\\F}}, \beta^{\text{hyb}}_{\substack{Q\\F}}$, once the productions $W^{\text{sr}}_{R}, W^{\text{sr}}_{F}, Q^{\text{sr}}_{R}, Q^{\text{sr}}_{F}, W_{\text{hyb}}, Q_{\text{hyb}}$ and the primary energy consumptions $P^{\text{sr}}_{\substack{R\\W}}, P^{\text{sr}}_{\substack{F\\W}}, P^{\text{sr}}_{\substack{R\\Q}}, P^{\text{sr}}_{\substack{F\\Q}}, P^{\text{hyb}}_{R}, P^{\text{hyb}}_{F}$ are known.

Figs. 2–5 show the results of the analysis by showing for the various parameters the ratios of values obtained with the STALPR method to the corresponding values obtained with the SRSPR method, plotted as functions of the degree of penetration of hybrid cogeneration in the local area, represented by the variable $x = W_{\text{hyb}}/(W^{\text{sr}}_{F} + W_{\text{hyb}})$. To obtain the actual value of each variable on the various curves, it is therefore necessary to associate the corresponding value of the SRSPR method reported in the second and third columns of Table 2. The reference primary energy factors for the SRSPR method are taken equal to the average factors that existed in the local area before the introduction of hybrid cogeneration, therefore, STALPR and SRSPR allocations coincide for $x = 0$. In each Figure we compare the results for two values of the reference efficiency for (single-resource single-production) power production from fossil-fuels, namely, a conservative value $\eta^{\text{sr}}_{FW} = 0.38$ and a more up-to-date value $\eta^{\text{sr}}_{FW} = 0.55$.

In general, it can be observed that the differences between the STALPR and SRSPR methods grow as hybrid cogeneration takes on higher shares of the local energy production portfolio and becomes important in areas where their level of penetration is relatively high. For instance, as shown in Fig. 5b the highest deviation of the $\text{COP}^{\text{hyb}}_{\substack{R\\Q}}$ from the SRSPR method is +58% at $x = 1$, while the difference remains below 10% if the penetration of hybrid cogeneration remains below 30%.

Nonetheless, it is important to remark that the results shown in Figs. 2 to 5 are specifically referred to the particular values chosen for the parameters that characterize the local area assumed in this case study (they are recalled for clarity in each chart). Significant changes in both the values and profiles are indeed to be expected for different local area conditions that can be found in other realistic energy portfolios.

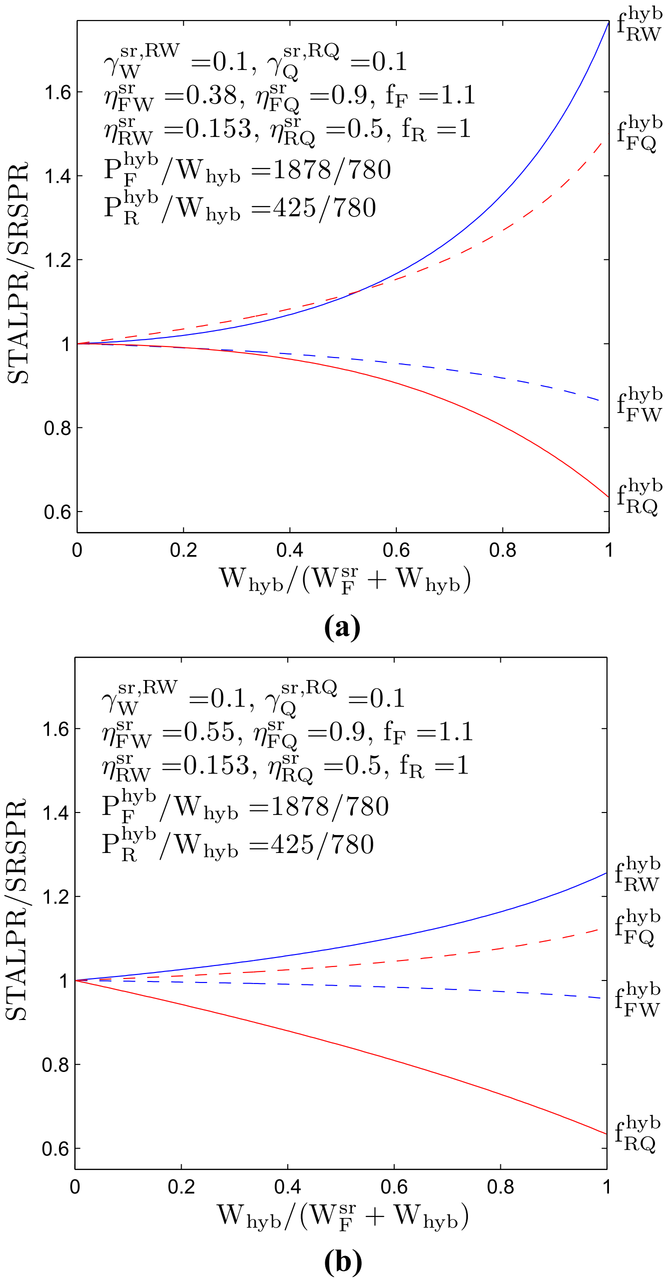

Fig. 2. Ratio of the STALPR to the SRSPR values for the partial primary energy factors $f^{\mathrm{hyb}}_{\mathrm{F}_{\mathrm{W}}}$, $f^{\mathrm{hyb}}_{\mathrm{F}_{\mathrm{Q}}}$, $f^{\mathrm{hyb}}_{\mathrm{R}_{\mathrm{W}}}$, $f^{\mathrm{hyb}}_{\mathrm{R}_{\mathrm{Q}}}$ of the hybrid cogeneration facility, plotted as functions of the hybrid penetration parameter $W_{\mathrm{hyb}}/(W^{\mathrm{sr}}_{\mathrm{F}} + W_{\mathrm{hyb}})$ for the parameters listed in Table 1 and (a) $\eta^{\mathrm{sr}}_{\mathrm{F}_{\mathrm{W}}} = 0.38$, (b) $\eta^{\mathrm{sr}}_{\mathrm{F}_{\mathrm{W}}} = 0.55$. The SRSPR values are given in Table 2 and are computed assuming reference values equal to the primary energy factors of the renewable-only fossil-only power plants in the area.

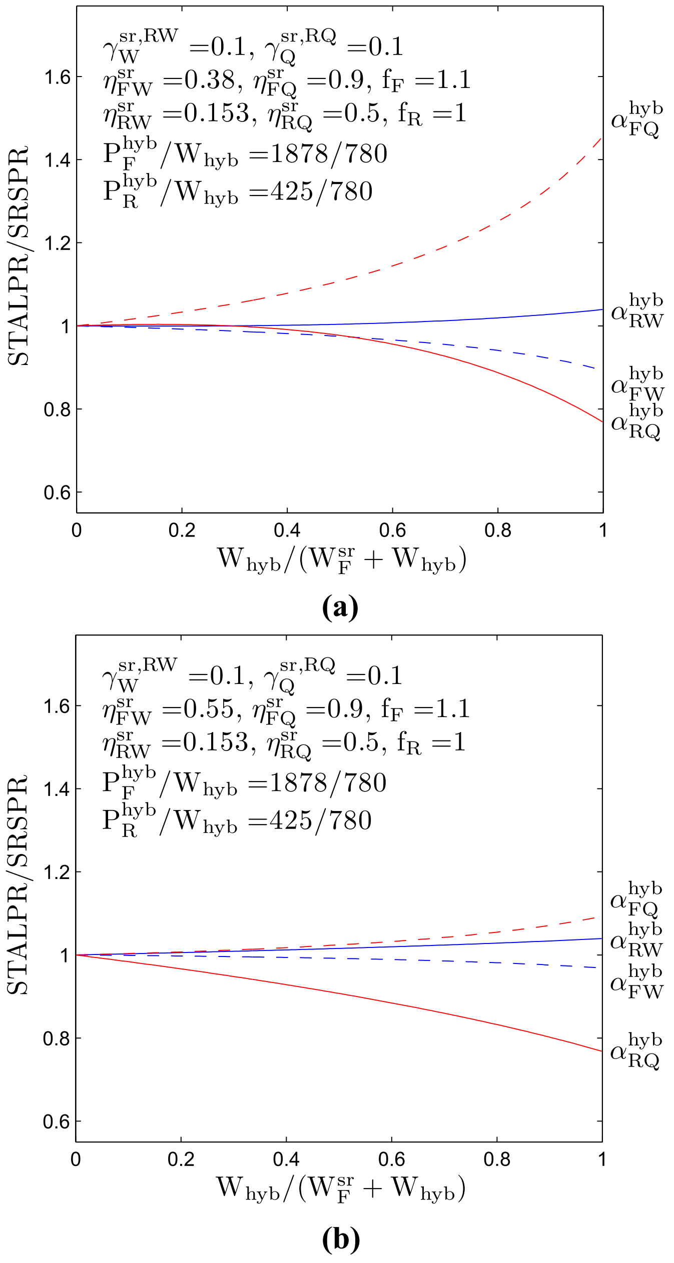

Fig. 3. Ratio of the STALPR to the SRSPR values for the resource allocation fractions $\alpha^{\mathrm{hyb}}_{\mathrm{F}_{\mathrm{W}}}$, $\alpha^{\mathrm{hyb}}_{\mathrm{F}_{\mathrm{Q}}}$, $\alpha^{\mathrm{hyb}}_{\mathrm{R}_{\mathrm{W}}}$, $\alpha^{\mathrm{hyb}}_{\mathrm{R}_{\mathrm{Q}}}$ of the hybrid cogeneration facility, plotted as functions of the hybrid penetration parameter $W_{\mathrm{hyb}}/(W^{\mathrm{sr}}_{\mathrm{F}} + W_{\mathrm{hyb}})$ for the parameters listed in Table 1 and (a) $\eta^{\mathrm{sr}}_{\mathrm{F}_{\mathrm{W}}} = 0.38$, (b) $\eta^{\mathrm{sr}}_{\mathrm{F}_{\mathrm{W}}} = 0.55$. The SRSPR values are given in Table 2 and are computed assuming reference values equal to the primary energy factors of the renewable-only fossil-only power plants in the area.

In principle, a better understanding of the effects of the various parameters of the STLARP method could be obtained following a systematic analytical study [of the system of twelve equations in twelve unknowns defined by Eqs. (3), (4) and (18)] along the same lines of the analysis we developed for the simpler cases in Refs. [13,15]. Here, however, the additional mathematical complexity of the two-resources/two-goods problem prevented us so far from obtaining useful general considerations on the effects of the various parameters. Nevertheless, the mathematical complexity can be reduced to some level as shown in the Appendix, where the system of twelve equations in twelve unknowns that defines the STALPR method is reduced to a much more tractable system of three equations in three unknowns that can be easily implemented numerically, even in a spreadsheet equipped with a numerical solver.

The method we propose could be used as a basis for regulations aiming at providing an allocation scheme that remains *permanently fair* as the result of it being adaptive and dynamically tied to the evolving local area production scenario, through the (evolving)

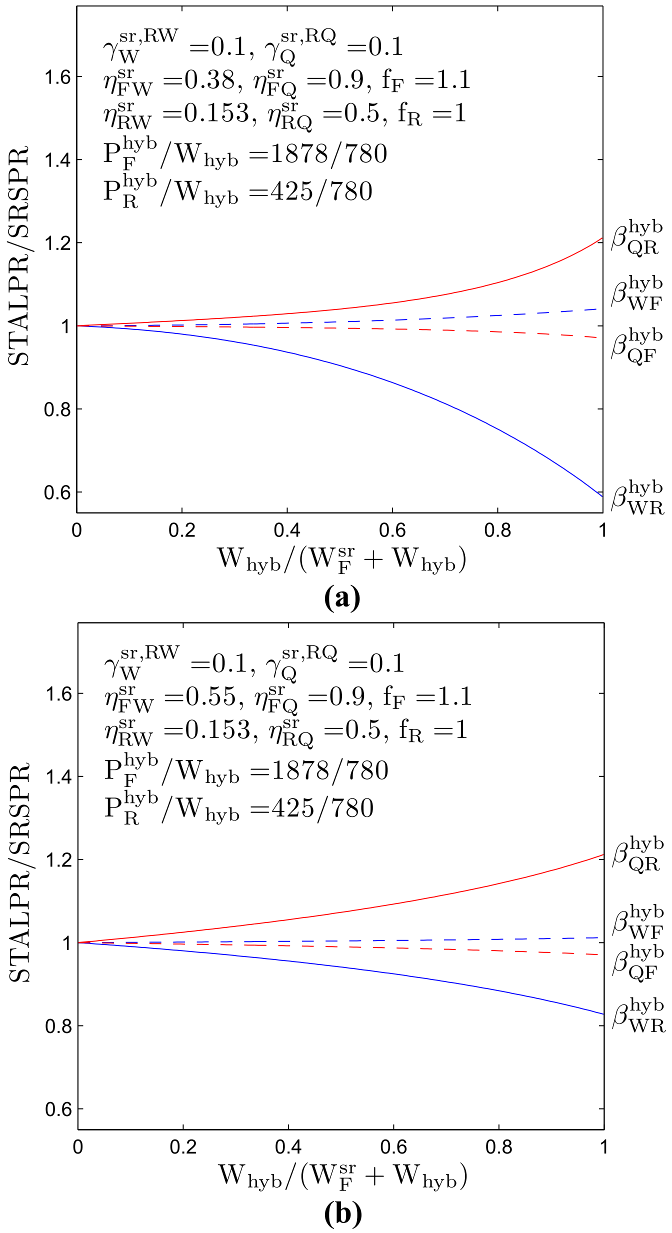

Fig. 4. Ratio of the STALPR to the SRSPR values for the product allocation fractions $\beta^{hyb}_{W_F}$, $\beta^{hyb}_{Q_F}$, $\beta^{hyb}_{W_R}$, $\beta^{hyb}_{Q_R}$ of the hybrid cogeneration facility, plotted as functions of the hybrid penetration parameter $W_{hyb}/(W^{sr}_F + W_{hyb})$ for the parameters listed in Table 1 and (a) $\eta^{sr}_{F_W} = 0.38$, (b) $\eta^{sr}_{F_W} = 0.55$. The SRSPR values are given in Table 2 and are computed assuming reference values equal to the primary energy factors of the renewable-only fossil-only power plants in the area.

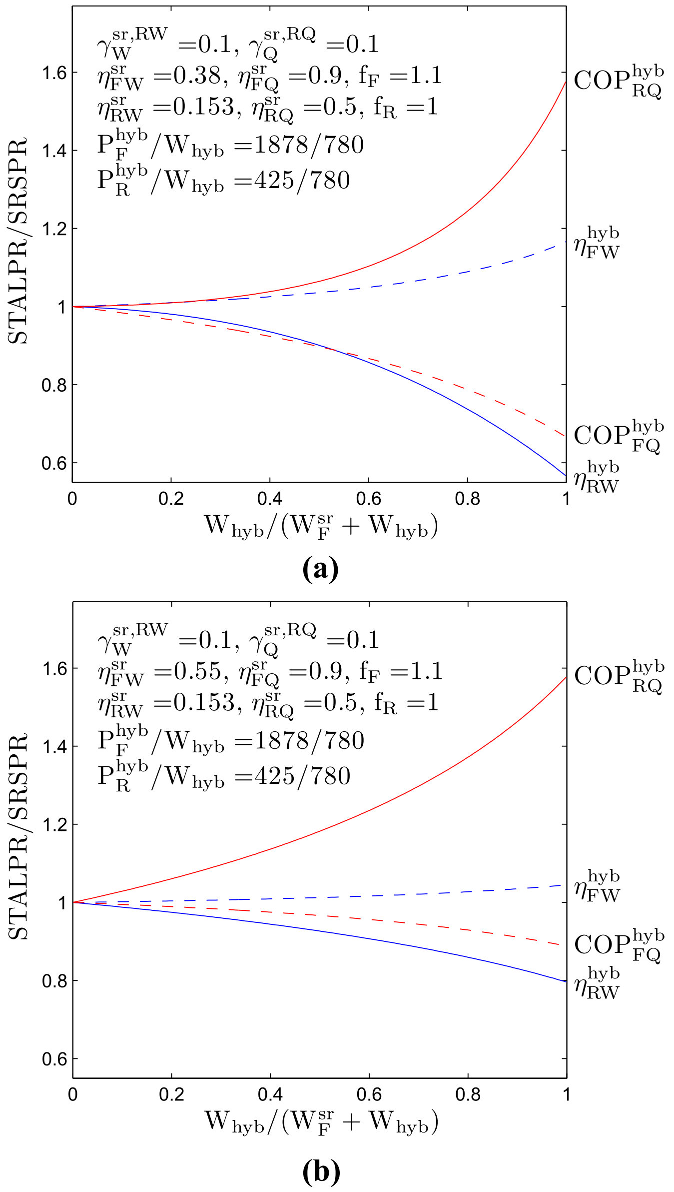

Fig. 5. Ratio of the STALPR to the SRSPR values for the partial energy conversion efficiencies $\eta^{hyb}_{F_W}$, $\mathrm{COP}^{hyb}_{F_Q}$, $\eta^{hyb}_{R_W}$, $\mathrm{COP}^{hyb}_{R_Q}$ of the hybrid cogeneration facility, plotted as functions of the hybrid penetration parameter $W_{hyb}/(W^{sr}_F + W_{hyb})$ for the parameters listed in Table 1 and (a) $\eta^{sr}_{F_W} = 0.38$, (b) $\eta^{sr}_{F_W} = 0.55$. The SRSPR values are given in Table 2 and are computed assuming reference values equal to the primary energy factors of the renewable-only fossil-only power plants in the area.

average conversion efficiencies of the local area, assuming these are known on a yearly basis. Such an implementation, of course, poses the important question of how to provide the utilities that must adopt the STALPR scheme with reliable data about the reference efficiencies of their local area, which are needed by the allocation scheme. To our knowledge, such reference conversion efficiencies can be extrapolated for each country from official energy statistics data, like those computed yearly for instance by the International Energy Agency [24] or British Petroleum [25]. On a more local scale, almost every country provides official data about its own internal energy balances (for instance in Italy, the electricity production by fuel is computed on a yearly basis by Terna [26]). Such data could represent a proper framework to characterize the local area of interest. Nonetheless, the objective of some energy policies may be to focus on more specific smaller areas (for instance, a region, a province, a county, a town, a district) affected by a lack of official reference data. Such cases may therefore require a more direct input by some local authority, for example, by collecting from every utility and energy consumer above a certain size the necessary data on their yearly consumptions and productions, so as to compute the average conversion efficiencies in an unambiguous way and publish the results so as to allow the implementation of the STALPR allocation method.

6. Partial primary energy savings and incentive policies

The allocation analysis discussed so far, may provide a sound framework to address incentive policy issues, because it yields the values of the efficiencies and coefficients of performance $\eta^{\rm hyb}_{F_W}$, ${\rm COP}^{\rm hyb}_{F_Q}$, $\eta^{\rm hyb}_{R_W}$, ${\rm COP}^{\rm hyb}_{R_Q}$ of the hybrid cogenerator. Within the process of quantifying the incentives for such facilities, these values are important because they can indeed be compared to the corresponding reference efficiencies that regulators may decide to set as threshold values, to be either prescribed periodically by some authority or taken to coincide with the yearly average values defined by the STALPR method for a local area or other reference portfolio to which the hybrid cogenerator belongs.

Notably, in this context it is convenient to introduce the partial primary energy savings that can be achieved by employing hybrid cogenerators instead of single-resource single-production facilities that use same fuel and renewable resource, operating with a prescribed set of references efficiencies and coefficients of performance $\eta^{\rm ref}_{F_W}$, ${\rm COP}^{\rm ref}_{F_Q}$, $\eta^{\rm ref}_{R_W}$, ${\rm COP}^{\rm ref}_{R_Q}$.

Following this approach, for a hybrid cogenerator we can define the following partial primary energy savings for the conversion of fossil-fuel energy to electricity

$$\mathrm{PES}^{\rm hyb}_{F_W}=\frac{f^{\rm hyb}_F\beta^{\rm hyb}_{W_F}W_{\rm hyb}/\eta^{\rm ref}_{F_W}-\alpha^{\rm hyb}_{F_W}P^{\rm hyb}_F}{f^{\rm hyb}_F\beta^{\rm hyb}_{W_F}W_{\rm hyb}/\eta^{\rm ref}_{F_W}}=1-\frac{\eta^{\rm ref}_{F_W}}{\eta^{\rm hyb}_{F_W}} \tag{19}$$

for the conversion of fossil-fuel energy to heat,

$$\mathrm{PES}^{\rm hyb}_{F_Q}=\frac{f^{\rm hyb}_F\beta^{\rm hyb}_{Q_F}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{F_Q}-\alpha^{\rm hyb}_{F_Q}P^{\rm hyb}_F}{f^{\rm hyb}_F\beta^{\rm hyb}_{Q_F}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{F_Q}}=1-\frac{\mathrm{COP}^{\rm ref}_{F_Q}}{\mathrm{COP}^{\rm hyb}_{F_Q}} \tag{20}$$

for the conversion of renewable-resource energy to electricity,

$$\mathrm{PES}^{\rm hyb}_{R_W}=\frac{f^{\rm hyb}_R\beta^{\rm hyb}_{W_R}W_{\rm hyb}/\eta^{\rm ref}_{R_W}-\alpha^{\rm hyb}_{R_W}P^{\rm hyb}_R}{f^{\rm hyb}_R\beta^{\rm hyb}_{W_R}W_{\rm hyb}/\eta^{\rm ref}_{R_W}}=1-\frac{\eta^{\rm ref}_{R_W}}{\eta^{\rm hyb}_{R_W}} \tag{21}$$

and for the conversion of renewable-resource energy to heat,

$$\mathrm{PES}^{\rm hyb}_{R_Q}=\frac{f^{\rm hyb}_R\beta^{\rm hyb}_{Q_R}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{R_Q}-\alpha^{\rm hyb}_{R_Q}P^{\rm hyb}_R}{f^{\rm hyb}_R\beta^{\rm hyb}_{Q_R}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{R_Q}}=1-\frac{\mathrm{COP}^{\rm ref}_{R_Q}}{\mathrm{COP}^{\rm hyb}_{R_Q}} \tag{22}$$

Moreover, we can define the overall primary fossil-fuel energy savings,

$$\mathrm{PES}^{\rm hyb}_F=1-\frac{P^{\rm hyb}_F}{f^{\rm hyb}_F\beta^{\rm hyb}_{W_F}W_{\rm hyb}/\eta^{\rm ref}_{F_W}+f^{\rm hyb}_F\beta^{\rm hyb}_{Q_F}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{F_Q}} \tag{23}$$

the overall renewable primary energy savings,

$$\mathrm{PES}^{\rm hyb}_R=1-\frac{P^{\rm hyb}_R}{f^{\rm hyb}_R\beta^{\rm hyb}_{W_R}W_{\rm hyb}/\eta^{\rm ref}_{R_W}+f^{\rm hyb}_R\beta^{\rm hyb}_{Q_R}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{R_Q}} \tag{24}$$

the fossil + renewable primary energy savings of electricity production,

$$\mathrm{PES}^{\rm hyb}_W=1-\frac{\alpha^{\rm hyb}_{F_W}P^{\rm hyb}_F+\alpha^{\rm hyb}_{R_W}P^{\rm hyb}_R}{f^{\rm hyb}_F\beta^{\rm hyb}_{W_F}W_{\rm hyb}/\eta^{\rm ref}_{F_W}+f^{\rm hyb}_R\beta^{\rm hyb}_{W_R}W_{\rm hyb}/\eta^{\rm ref}_{R_W}} \tag{25}$$

and the fossil + renewable primary energy savings of heat production,

$$\mathrm{PES}^{\rm hyb}_Q=1-\frac{\alpha^{\rm hyb}_{F_Q}P^{\rm hyb}_F+\alpha^{\rm hyb}_{R_Q}P^{\rm hyb}_R}{f^{\rm hyb}_F\beta^{\rm hyb}_{Q_F}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{F_Q}+f^{\rm hyb}_R\beta^{\rm hyb}_{Q_R}Q_{\rm hyb}/\mathrm{COP}^{\rm ref}_{R_Q}} \tag{26}$$

Thus, for instance, policies intended to promote an efficient utilization of either the fossil or renewable sources should prescribe positive values (or above a certain threshold) of $\mathrm{PES}^{\rm hyb}_F$ and $\mathrm{PES}^{\rm hyb}_R$ respectively.

It is easy to verify that the above definitions are interrelated as follows

$$\left(\frac{\beta^{\rm hyb}_{W_F}W_{\rm hyb}}{\eta^{\rm ref}_{F_W}}+\frac{\beta^{\rm hyb}_{Q_F}Q_{\rm hyb}}{\mathrm{COP}^{\rm ref}_{F_Q}}\right)\mathrm{PES}_F=\frac{\beta^{\rm hyb}_{W_F}W_{\rm hyb}}{\eta^{\rm ref}_{F_W}}\mathrm{PES}_{F_W}+\frac{\beta^{\rm hyb}_{Q_F}Q_{\rm hyb}}{\mathrm{COP}^{\rm ref}_{F_Q}}\mathrm{PES}_{F_Q} \tag{27}$$

$$\left(\frac{\beta^{\rm hyb}_{W_R}W_{\rm hyb}}{\eta^{\rm ref}_{R_W}}+\frac{\beta^{\rm hyb}_{Q_R}Q_{\rm hyb}}{\mathrm{COP}^{\rm ref}_{R_Q}}\right)\mathrm{PES}_R=\frac{\beta^{\rm hyb}_{W_R}W_{\rm hyb}}{\eta^{\rm ref}_{R_W}}\mathrm{PES}_{R_W}+\frac{\beta^{\rm hyb}_{Q_R}Q_{\rm hyb}}{\mathrm{COP}^{\rm ref}_{R_Q}}\mathrm{PES}_{R_Q} \tag{28}$$

$$\left(\frac{f^{\rm hyb}_F\beta^{\rm hyb}_{W_F}}{\eta^{\rm ref}_{F_W}}+\frac{f^{\rm hyb}_R\beta^{\rm hyb}_{W_R}}{\eta^{\rm ref}_{R_W}}\right)\mathrm{PES}_W=\frac{f^{\rm hyb}_F\beta^{\rm hyb}_{W_F}}{\eta^{\rm ref}_{F_W}}\mathrm{PES}_{F_W}+\frac{f^{\rm hyb}_R\beta^{\rm hyb}_{W_R}}{\eta^{\rm ref}_{R_W}}\mathrm{PES}_{R_W} \tag{29}$$

$$\left(\frac{f^{\rm hyb}_F\beta^{\rm hyb}_{Q_F}}{\mathrm{COP}^{\rm ref}_{F_Q}}+\frac{f^{\rm hyb}_R\beta^{\rm hyb}_{Q_R}}{\mathrm{COP}^{\rm ref}_{R_Q}}\right)\mathrm{PES}_Q=\frac{f^{\rm hyb}_F\beta^{\rm hyb}_{Q_F}}{\mathrm{COP}^{\rm ref}_{F_Q}}\mathrm{PES}_{F_Q}+\frac{f^{\rm hyb}_R\beta^{\rm hyb}_{Q_R}}{\mathrm{COP}^{\rm ref}_{R_Q}}\mathrm{PES}_{R_Q} \tag{30}$$

Table 3 lists the resulting partial energy savings calculated according to Eqs. (19–26) as resulting from the SRSPR and ExRR

Table 3
PES (Primary energy savings) as resulting from the SRSPR and ExRR methods, for the hybrid solar & fossil/heat & electricity cogeneration case study defined in Section 3 (parameters summarized in Table 1).

	SRSPR		ExRR	
	$\eta^{sr}_{\substack{F\\W}} = 0.38$	$\eta^{sr}_{\substack{F\\W}} = 0.55$	$T^{hyb}_{Q} = 95°C$	$T^{hyb}_{Q} = 150°C$
$PES^{hyb}_{\substack{F\\W}}$	0.2590	0.0395	−1.3958	−1.2753
$PES^{hyb}_{\substack{F\\Q}}$	0.2333	−0.0226	−1.3969	−1.2762
$PES^{hyb}_{\substack{R\\W}}$	0.2182	−0.0999	−1.6353	−1.5027
$PES^{hyb}_{\substack{R\\Q}}$	0.4137	0.4137	−1.6366	−1.5038
PES^{hyb}_{F}	0.2542	0.0244	−1.3961	−1.2755
PES^{hyb}_{R}	0.2542	0.0248	−1.6351	−1.5026
PES^{hyb}_{W}	0.2514	0.0137	−1.4368	−1.3142
PES^{hyb}_{Q}	0.2667	0.0580	−1.4375	−1.3149

methods. It can be observed that the SRSPR approach is capable to feature the benefits of both hybridization and cogeneration, from the point of view of the efficient conversion of the renewable and fossil fuel (positive values of PES^{hyb}_{R} and PES^{hyb}_{F}) as well as of the efficient overall production of electricity and heat (positive values of PES^{hyb}_{W} and PES^{hyb}_{Q}). On the contrary, the negative values of PES obtained with the ExRR method (which takes as reference the efficiencies of thermodynamically reversible machinery) confirm that this approach is unfair in the present technological context.

7. Generalization of the SRSPR and STALPR allocation methods to multi-resource/multi-generation

In the present section we generalize the SRSPR and STALPR methods to a local-area scenario including multi-generation facilities producing a mix of different *goods* by consuming a mix of *resources* or *groups of resources*. Since our main interest is in energy production and consumption, typical examples of *goods* can be: electric power, process heat at different temperature levels, district heating, compressed air, chilling or refrigeration, water desalination as well as other energy-intensive productions such as cement, steel, aluminum, other materials or flows. Similarly in the energy production and consumption context, examples of *resources* can be: oil, natural gas, coal, nuclear energy fuels, biomass, hydroelectric, solar, wind, tidal and geothermal energy, while examples of homogenous *groups of resources* can be the renewable and non renewable energy resources but the same reasoning can be applied as well to materials processing or other industrial facilities where in addition to energy resources we must also account for materials (limestone, iron ore, bauxite and other primary raw materials).

We consider a local area of interest that includes multi-generation facilities that use and transform a mix of different resources. Let the generic term *primary impact* identify the particular effect or impact that we wish to quantify and allocate in a fair way among the various goods produced in a local area by quantifying for each good the relative contribution of impact that is to be associated with each different resource or group of resources. Examples of *primary impacts* are:

- primary energy consumption
- greenhouse gas (CO_2 equivalent) emissions
- exergy consumption (by irreversibility)

As a result, particularly interesting in multi-resource CHP (heat and power cogeneration) are the following *primary impact-goods-resources* combinations:

- (primary energy consumption)-(heat and electric power)-(renewable and non-renewable resources)
- (CO_2 equivalent emissions)-(heat and electric power)-(renewable and non-renewable resources)

but many more combinations may also be important.

7.1. General formulation

Let us consider the *k*-th production facility in a local area of interest. We denote by $G^{\text{facility k}}_{\text{product j}}$ the “amount of good (product)” of *j*-th type it produces and by $R^{\text{facility k}}_{\text{resource i}}$ the “amount of resource” of *i*-th type it consumes. We use different letters to denote the “amounts” of goods and resources because we are free to select different methods of accounting (and hence valueing). Indeed, the energy resources (fossil fuels, solar energy, etc.) need not be necessarily expressed and accounted for only in terms of their energy content. For example, they could also be valued in terms of their exergies or their economic market values, or the amount of primary energy consumption or greenhouse gas production associated with their consumption on a well-to-waste full-life-cycle basis. Similarly, the raw materials (iron ore, etc.) as well as the produced goods (electricity, heat, steel, cement, potable water, etc.) need not be necessarily expressed and accounted for only in terms of their amounts in the most natural units (chilowatthours, tons, etc.), but we could also value them in terms of their exergies or their economic market values, or some measure of the environmental fingerprint associated with their well-to-waste life cycle. The quantities $\gamma^{\text{facility k}}_{\text{product j}}$, $\rho^{\text{facility k}}_{\text{product j}}$, $\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}$, $\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}$, $\varphi^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}$, $\varphi^{\text{loc}}_{\substack{\text{resource i}\\ \text{product j}}}$ defined below are independent of this choice of accounting method as long as all facilities use the same accounting units for the corresponding resources and goods. Thus, these quantities represent a ‘universal’ aspect of the allocation problem which is independent of such choice. However, the quantities $f^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}$, $f^{\text{local}}_{\substack{\text{resource i}\\ \text{product j}}}$ which are needed as intermediate variables in the solution of the allocation problem do depend on the chosen set of accounting units, and so do the results as well as the very meaning of the allocation problem itself. For instance, if for the hybrid renewable & fossil/heat & electricity cogenerator considered in the previous section, instead of accounting resources in terms of their respective primary energy impacts we do it in terms of their respective greenhouse-gases fingerprints, we obtain an entirely different allocation problem. However, it can be noted that the two problems are related through the ‘universal’ part of the solution of the allocation problem which we will see below depends only on the market shares $\gamma^{\text{facility k}}_{\text{product j}}$ and $\rho^{\text{facility k}}_{\text{product j}}$ which characterize the local-area scenario.

We begin by defining the local market share of facility *k* with respect to the production of good *j*

$$\gamma^{\text{facility k}}_{\text{product j}} = \frac{G^{\text{facility k}}_{\text{product j}}}{G^{\text{loc,overall}}_{\text{product j}}} \quad \text{where} \quad G^{\text{loc,overall}}_{\text{product j}} = \sum_{n} G^{\text{facility n}}_{\text{product j}} \tag{31}$$

and the local market share of facility *k* with respect to the consumption of resource *i*

$$\rho^{\text{facility k}}_{\text{resource i}} = \frac{R^{\text{facility k}}_{\text{resource i}}}{R^{\text{loc,overall}}_{\text{resource i}}} \quad \text{where} \quad R^{\text{loc,overall}}_{\text{resource i}} = \sum_n R^{\text{facility n}}_{\text{resource i}} \tag{32}$$

Solving the allocation problem for these facilities means to identify the quantities $R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}$ and $G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}$, where $R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}$ is the fair share of consumption of resource *i* in facility *k* that can be attributed to the production of good *j* and $G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}$ is the fair share of production of good *j* in facility *k* that can be attributed to the consumption of resource *i*. Of course,

$$R^{\text{facility k}}_{\text{resource i}} = \sum_j R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} \quad \text{and} \quad G^{\text{facility k}}_{\text{product j}} = \sum_i G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} \tag{33}$$

and $R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = R^{\text{facility k}}_{\text{resource i}}$ if $N_{\text{products}} = 1$, $G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = G^{\text{facility k}}_{\text{product j}}$ if $N_{\text{resources}} = 1$. These quantities, for $R^{\text{facility k}}_{\text{resource i}} \neq 0$ define the resource allocation fractions

$$\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}}{R^{\text{facility k}}_{\text{resource i}}} \tag{34}$$

and for $G^{\text{facility k}}_{\text{product j}} \neq 0$ the product allocation fractions

$$\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = \frac{G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}}{G^{\text{facility k}}_{\text{product j}}} \tag{35}$$

where, clearly, $\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = 1$ if $N_{\text{products}} = 1$ and $\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = 1$ if $N_{\text{resources}} = 1$. It is also be convenient to conventionally set $\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = 0$ for every *j* whenever $R^{\text{facility k}}_{\text{resource i}} = 0$, i.e., when facility *k* does not consume resource *i*, and $\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = 0$ for every *i* whenever $G^{\text{facility k}}_{\text{product j}} = 0$, i.e., facility *k* does not produce good *j*.

Next, we define the partial resource-*i*-to-good-*j* factors for facility *k*

$$f^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}}{G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}} = \frac{\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} R^{\text{facility k}}_{\text{resource i}}}{\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} G^{\text{facility k}}_{\text{product j}}}$$

$$= \frac{\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} \rho^{\text{facility k}}_{\text{resource i}} R^{\text{loc,overall}}_{\text{resource i}}}{\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} \gamma^{\text{facility k}}_{\text{product j}} G^{\text{loc,overall}}_{\text{product j}}} \tag{36}$$

and we normalize them by the ratio of the overall consumption of resources to the overall production of goods in the local area

$$\phi^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{G^{\text{loc,overall}}_{\text{product j}}}{R^{\text{loc,overall}}_{\text{resource i}}} f^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} \rho^{\text{facility k}}_{\text{resource i}}}{\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} \gamma^{\text{facility k}}_{\text{product j}}} \tag{37}$$

Similarly, we define the local-area-averaged partial resource-*i*-to-good-*j* factors for the local area

$$f^{\text{loc}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{\sum_k R^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}}{\sum_k G^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}} \tag{38}$$

and we normalize them in the same way

$$\phi^{\text{loc}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{G^{\text{loc,overall}}_{\text{product j}}}{R^{\text{loc,overall}}_{\text{resource i}}} f^{\text{loc}}_{\substack{\text{resource i}\\ \text{product j}}} \tag{39}$$

Finally, according to the STALPR method, the allocation fractions based on the local-area-averaged resource-*i*-to-good-*j* ratios are adopted as follows:

- the key assumption for the resource allocation fractions is

$$\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{f^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product j}}} G^{\text{facility k}}_{\text{product j}}}{\sum_m f^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product m}}} G^{\text{facility k}}_{\text{product m}}} \tag{40}$$

meaning that the consumption of resources in each facility are allocated among the different goods it produces based on the relative proportions of the consumptions of the respective resource that would be required if the same amounts of goods were to be produced in single-resource separate-production facilities operating with resource-to-product ratios equal to the respective average resource-to-product ratios characterizing the local area of interest; and

- the key assumption for the product allocation fractions is

$$\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = \frac{\dfrac{R^{\text{facility k}}_{\text{resource i}}}{f^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product j}}}}}{\sum_n \dfrac{R^{\text{facility k}}_{\text{resource n}}}{f^{\text{loc.ave.}}_{\substack{\text{resource n}\\ \text{product j}}}}} \tag{41}$$

meaning that the production of goods in each facility are allocated among the different resources it consumes based on the relative proportions of the productions of the respective good that would be obtained if the same amounts of resources were to be used in single-resource separate-production facilities operating with resource-to-product ratios equal to the respective average resource-to-product ratios characterizing the local area of interest.

By combining the above equations, we readily obtain the following system of $(2N_{\text{facilities}} + 1)N_{\text{resources}}N_{\text{products}}$ independent coupled equations

$$
\begin{aligned}
&\phi^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{\sum_k \alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}\, \rho^{\text{facility k}}_{\text{resource i}}}{\sum_k \beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}\, \gamma^{\text{facility k}}_{\text{product j}}}\\
&\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{\phi^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product j}}}\, \gamma^{\text{facility k}}_{\text{product j}}}{\sum_m \phi^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product m}}}\, \gamma^{\text{facility k}}_{\text{product m}}} \quad \text{if } \rho^{\text{facility k}}_{\text{resource i}} \neq 0\\
&\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = \frac{\dfrac{\rho^{\text{facility k}}_{\text{resource i}}}{\phi^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product j}}}}}{\sum_n \dfrac{\rho^{\text{facility k}}_{\text{resource n}}}{\phi^{\text{loc.ave.}}_{\substack{\text{resource n}\\ \text{product j}}}}} \quad \text{if } \gamma^{\text{facility k}}_{\text{product j}} \neq 0\\
&\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = 0 \quad \text{if } \rho^{\text{facility k}}_{\text{resource i}} = 0\\
&\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}} = 0 \quad \text{if } \gamma^{\text{facility k}}_{\text{product j}} = 0
\end{aligned}
\tag{42}
$$

which, for given values of the local market shares $\gamma^{\text{facility k}}_{\text{product j}}$ and $\rho^{\text{facility k}}_{\text{resource i}}$ which characterize the local-area scenario, can be solved numerically for the $(2N_{\text{facilities}}+1)N_{\text{resources}}N_{\text{products}}$ STALPR unknowns $\varphi^{\text{loc.ave.}}_{\substack{\text{resource i}\\ \text{product j}}}$, $\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}$, $\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}$. Of course, once the values of these unknowns are obtained, all other quantities may be computed, for example, the partial resource-*i*-to-good-*j* factors for facility *k* are obtained from Eq. (36), i.e.,

$$
f^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{\alpha^{\text{facility k}}_{\substack{\text{resource i}\\ \text{product j}}}\, \rho^{\text{facility k}}_{\text{resource i}}}{\beta^{\text{facility k}}_{\substack{\text{product j}\\ \text{resource i}}}\, \gamma^{\text{facility k}}_{\text{product j}}}\, \frac{R^{\text{loc,overall}}_{\text{resource i}}}{G^{\text{loc,overall}}_{\text{product j}}}
\tag{43}
$$

where it can be noted that the first fraction on the rhs is the ‘universal’ part of the solution whereas the second fraction brings in the particular set of accounting choices which characterize the allocation problem.

It is finally noteworthy that since the system of Eq. (42) must be solved numerically, it may be convenient to adopt as initial guess the solution of the SRSPR method for prescribed reference values equal to the average primary energy factors of the single-resource single-product facilities in the local area, i.e., by setting $f^{\text{ref}}_{\substack{\text{resource i}\\ \text{product j}}} = \overline{f}^{\text{SRSPfacilities}}_{\substack{\text{resource i}\\ \text{product j}}}$. If for some given resource/product pair *ij* there is no single-resource/single-product facility in the local area, then we need to adopt some reasonable reference $f^{\text{ref}}_{\substack{\text{resource i}\\ \text{product j}}}$. In the multi-resource/multi-product case, the nonzero resource and product SRSPR allocation fractions are

$$
\begin{aligned}
&\alpha^{\substack{\text{SRSPR}\\ \text{facility k}}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{f^{\text{ref}}_{\substack{\text{resource i}\\ \text{product j}}}\, G^{\text{facility k}}_{\text{product j}}}{\sum_m f^{\text{ref}}_{\substack{\text{resource i}\\ \text{product m}}}\, G^{\text{facility k}}_{\text{product m}}} \quad \text{if } R^{\text{facility k}}_{\text{resource i}} \neq 0\\
&\beta^{\substack{\text{SRSPR}\\ \text{facility k}}}_{\substack{\text{product j}\\ \text{resource i}}} = \frac{\dfrac{R^{\text{facility k}}_{\text{resource i}}}{f^{\text{ref}}_{\substack{\text{resource i}\\ \text{product j}}}}}{\sum_n \dfrac{R^{\text{facility k}}_{\text{resource n}}}{f^{\text{ref}}_{\substack{\text{resource n}\\ \text{product j}}}}} \quad \text{if } G^{\text{facility k}}_{\text{product j}} \neq 0
\end{aligned}
\tag{44}
$$

Therefore, a convenient initial guess for the STALPR system of Eq. (42) is, assuming for simplicity that the local area has at least a single-resource single-product facility for each resource–product pair *ij*,

$$
\begin{aligned}
&\phi^{\substack{\text{GUESS}\\ \text{loc.ave.}}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{G^{\text{loc,overall}}_{\text{product j}}}{R^{\text{loc,overall}}_{\text{resource i}}}\, \overline{f}^{\text{SRSPfacilities}}_{\substack{\text{resource i}\\ \text{product j}}}\\
&\alpha^{\substack{\text{GUESS}\\ \text{facility k}}}_{\substack{\text{resource i}\\ \text{product j}}} = \frac{\phi^{\substack{\text{GUESS}\\ \text{loc.ave.}}}_{\substack{\text{resource i}\\ \text{product j}}}\, \gamma^{\text{facility k}}_{\text{product j}}}{\sum_m \phi^{\substack{\text{GUESS}\\ \text{loc.ave.}}}_{\substack{\text{resource i}\\ \text{product m}}}\, \gamma^{\text{facility k}}_{\text{product m}}} \quad \text{if } \rho^{\text{facility k}}_{\text{resource i}} \neq 0\\
&\beta^{\substack{\text{GUESS}\\ \text{facility k}}}_{\substack{\text{product j}\\ \text{resource i}}} = \frac{\dfrac{\rho^{\text{facility k}}_{\text{resource i}}}{\phi^{\substack{\text{GUESS}\\ \text{loc.ave.}}}_{\substack{\text{resource i}\\ \text{product j}}}}}{\sum_n \dfrac{\rho^{\text{facility k}}_{\text{resource n}}}{\phi^{\substack{\text{GUESS}\\ \text{loc.ave.}}}_{\substack{\text{resource n}\\ \text{product j}}}}} \quad \text{if } \gamma^{\text{facility k}}_{\text{product j}} \neq 0
\end{aligned}
\tag{45}
$$

8. Conclusions

In this paper, we consider a multi-resource multi-generation facility and address the problem of allocating each resource among the different products and each product among the different resources. The aim of the allocation problem is to provide a reasonable and general criterion to define what quota of each the resources consumed by the facility is to be attributed to each of the goods it produces and what quota of each of the goods it produces is to be attributed to each of the resources it consumes. For a hybrid cogenerator producing heat and power by consuming a mix of a renewable resource and a fossil-fuel the allocation problem provides answers to questions such as: how much of the produced electricity and heat are renewable? What is the efficiency of production of electricity from fossil fuel in the cogenerator? What is the coefficient of performance of the production of heat in the cogenerator? These questions represent important aspects of the considerations about energy savings, avoided greenhouse emissions, efficient use and preservation of fossil resources, and promotion of renewable resources that regulators must take into account when designing energy policies and defining incentives in current scenarios where increasing and renewed attention is reserved to the use of renewable resources in hybrid cogeneration technologies.

The present paper completes and generalizes two previous studies that we published on allocation, namely Ref. [13], where we focus on the (single-resource/two-products) allocation of the primary energy consumption in a cogeneration facility among its heat and power production, and Ref. [15] where we focus on the (two-resources/single-product) allocation of the electricity production in a hybrid power plant among its solar energy and fossil fuel consumption.

To illustrate quantitatively the allocation problem, we refer to the realistic case of hybrid cogeneration facilities based on the technology of SICCS (Solar-Integrated Combined-Cycle System) operated in cogeneration mode, included in a generic local area with other traditional single-resource separate-production heat and power facilities.

We first extend to hybrid cogeneration the main classical allocation methods, namely, the SRSPR (Single Resource Separate Production Reference method) and the ExRR (Exergy-based Reversible-Reference method). It is shown that the SRSPR can be considered a fair approach, capable in principle to allot the benefits of hybridization and cogeneration in a fair way among the resources and products, provided the reference single-resource/single-production efficiencies on which it is based are not too different from the actual average efficiencies of the energy portfolio in the local area with which the hybrid cogenerator is to be compared. On the contrary, the ExRR (which takes as reference single-resource/single-production efficiencies those of thermodynamically reversible machinery) is unfair in the present technological context because, as we already noted in Refs. [13,15], current fossil-fuel and power production technologies have average second-law efficiencies much closer to 100% than current heat production and renewable technologies. Thus, compared with the SRSPR method, the ExRR method credits the thermal energy with too high a share of the cogeneration benefits leaving an unfairly little share of the primary energy savings to the cogenerated electricity. Moreover, it credits the renewable resources with too high a share of the hybridization benefits attributing an unfairly little share of the production to the co-consumed fossil fuel.

Nonetheless, the SRSPR method still has a limit in that it requires a set of prescribed reference primary energy conversion efficiencies to be defined every now and then by some authority, but it does not specify criteria for the choice of such efficiencies, with the problem that if they differ from the actual values that characterize the energy conversion portfolio of the local area where the hybrid cogenerators are included they result in unfair comparisons which in turn may induce possibly distorted incentive policies thus defeating their objectives.

Therefore, we formulate and extend to the present two-resources/two-products problem, the novel more consistent STALPR (Self-Tuned-Average-Local-Productions-Reference) method introduced in Refs. [13,15], whereby the allocation is adaptive and self-tuned to the actual efficiencies of the local energy scenario, with no need for prescribed reference efficiencies.

By applying the STALPR method to the considered hybrid cogenerator case study, it turns out that the differences with the SRSPR method grow as hybrid cogeneration technologies take higher shares of the local energy production portfolio and become important in areas where their penetration reaches relatively high levels.

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Appendix

A general formulation can be obtained by recasting the allocation problem defined in Section 2 in terms of the following new variables.

$$\sigma_{\mathrm{hyb}} = \frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{P_{\mathrm{R}}^{\mathrm{hyb}}} \qquad \varepsilon_{\mathrm{hyb}} = \frac{W_{\mathrm{hyb}}}{Q_{\mathrm{hyb}}} \tag{A.1}$$

$$\chi_{\mathrm{W}}^{\mathrm{hyb}} = \frac{\dfrac{\alpha^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{W}}}}{f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{W}}}}}{\dfrac{\alpha^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{W}}}}{f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{W}}}}} \qquad \chi_{\mathrm{Q}}^{\mathrm{hyb}} = \frac{\dfrac{\alpha^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{Q}}}}{f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{Q}}}}}{\dfrac{\alpha^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{Q}}}}{f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{Q}}}}} \tag{A.2}$$

$$\Phi_{\mathrm{F}}^{\mathrm{hyb}} = \frac{f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{Q}}}\,\beta^{\mathrm{hyb}}_{\substack{\mathrm{Q}\\ \mathrm{F}}}}{f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{W}}}\,\beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\ \mathrm{F}}}} \qquad \Phi_{\mathrm{R}}^{\mathrm{hyb}} = \frac{f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{Q}}}\,\beta^{\mathrm{hyb}}_{\substack{\mathrm{Q}\\ \mathrm{R}}}}{f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{W}}}\,\beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\ \mathrm{R}}}} \tag{A.3}$$

so that using Eqs. (1) and (5) the allocation fractions may be rewritten as follows

$$\alpha^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{W}}} = \frac{\varepsilon_{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{R}}^{\mathrm{hyb}}} \text{ and } \alpha^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{Q}}} = \frac{\Phi_{\mathrm{R}}^{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{R}}^{\mathrm{hyb}}} \tag{A.4}$$

$$\alpha^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{W}}} = \frac{\varepsilon_{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{F}}^{\mathrm{hyb}}} \text{ and } \alpha^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{Q}}} = \frac{\Phi_{\mathrm{F}}^{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{F}}^{\mathrm{hyb}}} \tag{A.5}$$

$$\beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\ \mathrm{R}}} = \frac{\chi_{\mathrm{W}}^{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}} \text{ and } \beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\ \mathrm{F}}} = \frac{\sigma_{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}} \tag{A.6}$$

$$\beta^{\mathrm{hyb}}_{\substack{\mathrm{Q}\\ \mathrm{R}}} = \frac{\chi_{\mathrm{Q}}^{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{Q}}^{\mathrm{hyb}}} \text{ and } \beta^{\mathrm{hyb}}_{\substack{\mathrm{Q}\\ \mathrm{F}}} = \frac{\sigma_{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{Q}}^{\mathrm{hyb}}} \tag{A.7}$$

and the partial primary energy factors as follows

$$f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{W}}} = \frac{\varepsilon_{\mathrm{hyb}}}{\chi_{\mathrm{W}}^{\mathrm{hyb}}}\,\frac{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{R}}^{\mathrm{hyb}}}\,\frac{P_{\mathrm{R}}^{\mathrm{hyb}}}{W_{\mathrm{hyb}}} \tag{A.8}$$

$$f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{W}}} = \frac{\varepsilon_{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}}}\,\frac{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{F}}^{\mathrm{hyb}}}\,\frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{W_{\mathrm{hyb}}} \tag{A.9}$$

$$f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\ \mathrm{Q}}} = \frac{\Phi_{\mathrm{R}}^{\mathrm{hyb}}}{\chi_{\mathrm{Q}}^{\mathrm{hyb}}}\,\frac{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{Q}}^{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{R}}^{\mathrm{hyb}}}\,\frac{P_{\mathrm{R}}^{\mathrm{hyb}}}{Q_{\mathrm{hyb}}} \tag{A.10}$$

$$f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\ \mathrm{Q}}} = \frac{\Phi_{\mathrm{F}}^{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}}}\,\frac{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{Q}}^{\mathrm{hyb}}}{\varepsilon_{\mathrm{hyb}} + \Phi_{\mathrm{F}}^{\mathrm{hyb}}}\,\frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{Q_{\mathrm{hyb}}} \tag{A.11}$$

The above formulation makes it apparent that in order to obtain the allocation fraction and the partial primary energy factors we must specify the values of χ_W^{hyb}, χ_Q^{hyb}, Φ_F^{hyb}, and Φ_R^{hyb}. Such specification constitutes the closure of the allocation problem and characterizes the allocation method. In particular according to this formulation each allocation method defined in Section 4 and 5 assumes the following general closure conditions.

$$\chi_W^{hyb} = \chi_W^{x} \quad \text{with } \chi_W^{x} = \frac{f^{x}_{F_W}}{f^{x}_{R_W}} \tag{A.12}$$

$$\chi_Q^{hyb} = \chi_Q^{x} \quad \text{with } \chi_Q^{x} = \frac{f^{x}_{F_Q}}{f^{x}_{R_Q}} \tag{A.13}$$

$$\Phi_F^{hyb} = \Phi_F^{x} \quad \text{with } \Phi_F^{x} = \frac{f^{x}_{F_Q}}{f^{x}_{F_W}} \tag{A.14}$$

$$\Phi_R^{hyb} = \Phi_R^{x} \quad \text{with } \Phi_R^{x} = \frac{f^{x}_{R_Q}}{f^{x}_{R_W}} \tag{A.15}$$

where for the SRSPR, ExRR and STALPR method, respectively, the superscript “x” in Eq. (A.12–A.15) stands for “ref”, “Ex” and “loc”.

It is noteworthy that for STLARP allocation, the problem can be simplified to a system of three equations in three unknowns chosen among χ_W^{hyb}, χ_Q^{hyb}, Φ_F^{hyb}, Φ_R^{hyb} as follows. First, we define the local market share of the single-resource separate-production facilities and of the hybrid facility with respect to the productions of electricity and heat.

$$\gamma^{sr}_{F_W} = \frac{W_F^{sr}}{W_F^{sr} + W_R^{sr} + W_{hyb}}, \quad \gamma^{sr}_{R_W} = \frac{W_R^{sr}}{W_F^{sr} + W_R^{sr} + W_{hyb}}, \quad \gamma_W^{hyb} = \frac{W_{hyb}}{W_F^{sr} + W_R^{sr} + W_{hyb}} \tag{A.16}$$

$$\gamma^{sr}_{F_Q} = \frac{Q_F^{sr}}{Q_F^{sr} + Q_R^{sr} + Q_{hyb}}, \quad \gamma^{sr}_{R_Q} = \frac{Q_R^{sr}}{Q_F^{sr} + Q_R^{sr} + Q_{hyb}}, \quad \gamma_Q^{hyb} = \frac{Q_{hyb}}{Q_F^{sr} + Q_R^{sr} + Q_{hyb}} \tag{A.17}$$

Then we define the local market share of the single-resource separate-production facilities and of the hybrid facility with respect to the consumption of fossil fuel and renewable energy.

$$\rho^{sr}_{F_W} = \frac{P^{sr}_{F_W}}{P^{sr}_{F_W} + P^{sr}_{F_Q} + P_F^{hyb}}, \quad \rho^{sr}_{F_Q} = \frac{P^{sr}_{F_Q}}{P^{sr}_{F_W} + P^{sr}_{F_Q} + P_F^{hyb}}, \quad \rho_F^{hyb} = \frac{P_F^{hyb}}{P^{sr}_{F_W} + P^{sr}_{F_Q} + P_F^{hyb}} \tag{A.18}$$

$$\rho^{sr}_{R_W} = \frac{P^{sr}_{R_W}}{P^{sr}_{R_W} + P^{sr}_{R_Q} + P_R^{hyb}}, \quad \rho^{sr}_{R_Q} = \frac{P^{sr}_{R_Q}}{P^{sr}_{R_W} + P^{sr}_{R_Q} + P_R^{hyb}}, \quad \rho_R^{hyb} = \frac{P_R^{hyb}}{P^{sr}_{R_W} + P^{sr}_{R_Q} + P_R^{hyb}} \tag{A.19}$$

Finally, we substitute Eqs. (A.4)–(A.7) into Eq. (18), using the above definitions and the STALPR closure relations, to obtain after some rearrangements the following four equations in the unknowns χ_W^{hyb}, χ_Q^{hyb}, Φ_F^{hyb}, Φ_R^{hyb},

$$\frac{\chi_W^{loc}}{\sigma_{loc}} = \frac{\rho^{sr}_{F_W} + \rho_F^{hyb}\frac{\varepsilon_{hyb}}{\varepsilon_{hyb}+\Phi_F^{loc}}}{\rho^{sr}_{R_W} + \rho_R^{hyb}\frac{\varepsilon_{hyb}}{\varepsilon_{hyb}+\Phi_R^{loc}}} \; \frac{\gamma^{sr}_{R_W} + \gamma_W^{hyb}\frac{\chi_W^{loc}}{\sigma_{hyb}+\chi_W^{loc}}}{\gamma^{sr}_{F_W} + \gamma_W^{hyb}\frac{\sigma_{hyb}}{\sigma_{hyb}+\chi_W^{loc}}} \tag{A.20}$$

$$\frac{\Phi_F^{loc}}{\varepsilon_{loc}} = \frac{\rho^{sr}_{F_Q} + \rho_F^{hyb}\frac{\Phi_F^{loc}}{\varepsilon_{hyb}+\Phi_F^{loc}}}{\rho^{sr}_{F_W} + \rho_F^{hyb}\frac{\varepsilon_{hyb}}{\varepsilon_{hyb}+\Phi_F^{hyb}}} \; \frac{\gamma^{sr}_{F_W} + \gamma_W^{hyb}\frac{\sigma_{hyb}}{\sigma_{hyb}+\chi_W^{hyb}}}{\gamma^{sr}_{F_Q} + \gamma_Q^{hyb}\frac{\sigma_{hyb}}{\sigma_{hyb}+\chi_Q^{loc}}} \tag{A.21}$$

$$\frac{\chi_Q^{loc}}{\sigma_{loc}} = \frac{\rho^{sr}_{F_Q} + \rho_F^{hyb}\frac{\Phi_F^{loc}}{\varepsilon_{hyb}+\Phi_F^{loc}}}{\rho^{sr}_{R_Q} + \rho_R^{hyb}\frac{\Phi_R^{loc}}{\varepsilon_{hyb}+\Phi_R^{loc}}} \; \frac{\gamma^{sr}_{R_Q} + \gamma_Q^{hyb}\frac{\chi_Q^{loc}}{\sigma_{hyb}+\chi_Q^{loc}}}{\gamma^{sr}_{F_Q} + \gamma_Q^{hyb}\frac{\sigma_{hyb}}{\sigma_{hyb}+\chi_Q^{loc}}} \tag{A.22}$$

$$\frac{\Phi_R^{loc}}{\varepsilon_{loc}} = \frac{\rho^{sr}_{R_Q} + \rho_R^{hyb}\frac{\Phi_R^{loc}}{\varepsilon_{hyb}+\Phi_R^{loc}}}{\rho^{sr}_{R_W} + \rho_R^{hyb}\frac{\varepsilon_{hyb}}{\varepsilon_{hyb}+\Phi_R^{loc}}} \; \frac{\gamma^{sr}_{R_W} + \gamma_W^{hyb}\frac{\chi_W^{loc}}{\sigma_{hyb}+\chi_W^{loc}}}{\gamma^{sr}_{R_Q} + \gamma_Q^{hyb}\frac{\chi_Q^{loc}}{\sigma_{hyb}+\chi_Q^{loc}}} \tag{A.23}$$

Only three of these equations are independent of one another because of the identity

$$\frac{\chi_W^{loc}\Phi_F^{loc}}{\chi_Q^{loc}\Phi_R^{loc}} = 1 \tag{A.24}$$

Therefore, we can use this identity to eliminate one of the four unknowns χ_W^{hyb}, χ_Q^{hyb}, Φ_F^{hyb}, Φ_R^{hyb} from three of the equations above and so we are left with a system of three equations in three unknowns.

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Glossary

$COP^k_{\substack{i\\j}}$ *and* COP^k_{ij}: partial energy-conversion efficiency of facility *k* from resource *i* to product *j*
E: energy
Ex: exergy
f: primary energy factor
$\bar{f}$: average primary energy factor
$G^{facility\ k}_{product\ j}$: amount of product of *j*-th type produced by facility *k*
$G^{facility\ k}_{\substack{product\ j\\resource\ i}}$: amount of product of *j*-th type produced by facility *k* considered as produced from the *i*-th type of resource
P: primary energy
$PES^k_{\substack{i\\j}}$: partial primary energy saving of facility *k* for the conversion from resource *i* to product *j*
Q: heat
$R^{facility\ k}_{resource\ i}$: amount of resource of *i*-th type consumed by facility *k*
$R^{facility\ k}_{\substack{resource\ i\\product\ j}}$: amount of resource of *i*-th type consumed by facility *k* considered as used to produce the *j*-th type of good
T: temperature
W: electricity

Superscripts

Ex: exergy
hyb: hybrid
loc: local area of interest
ref: reference
sr: single resource
x: generic allocation method

Subscripts

env: environment
F: fossil
hyb: hybrid
sr: single resource
Q: heat
R: renewable
W: electricity

Greek symbols

α: resource allocation fraction
β: product allocation fraction
$\gamma^{sr,RW}_{W}$: fraction of the overall electricity produced in the local area that comes from the renewable-only electricity facilities
$\gamma^{sr,RQ}_{Q}$: fraction of the overall heat produced in the local area that comes from the renewable-only heat facilities
$\gamma^{facility\ k}_{product\ j}$: dimensionless local market share detained by facility *k* in the local area scenario with respect to the production of good *j*, defined by Eq. (31)
ε_{hyb}: electric index, defined in Eq. (A.1)
$\varphi^{facility\ k}_{\substack{resource\ i\\product\ j}}$: dimensionless partial resource-*i*-to-good-*j* factor for facility *k*, defined by Eq. (37)
Φ^{hyb}_F *and* Φ^{hyb}_R: dimensionless parameters defined by Eq. (A.3)
$\eta^k_{\substack{i\\j}}$ *and* η^k_{ij}: partial energy-conversion efficiency of facility *k* from resource *i* to product *j*
$\rho^{facility\ k}_{resource\ i}$: dimensionless local market share detained by facility *k* in the local area scenario with respect to the consumption of resource *i*, defined by Eq. (32)
σ_{hyb}: fossil to renewable primary energy ratio, defined in Eq. (A.1)
χ^{hyb}_W *and* χ^{hyb}_Q: dimensionless parameters defined by Eq. (A.2)