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Electromagnetic Radiation: A Carrier of Energy and Entropy*

Starting from the properties of the electromagnetic radiation field at stable equilibrium, we derive expressions for the flows of energy and entropy between two black bodies at different temperatures, interacting only through electromagnetic radiation. We find that in general the interaction through radiation is nonwork but not heat. It is heat only if the temperature difference between the interacting systems is infinitesimal. [DOI: 10.1115/1.4026381]

1 Introduction

The electromagnetic field pervades all matter, fills the interatomic and intermolecular spaces, and interacts with atoms and molecules. As such it is one of the constituents of every physical system. Its properties are well known and experimentally verified [1–6]. Its stable equilibrium states provided the seminal ideas of modem physics. Its interactions with atoms and molecules are the subject of extensive experimental studies and the source of many recent technological developments.

In this paper we review the stable-equilibrium-state properties of the electromagnetic field, and discuss the model for a special class of nonequilibrium states that constitutes the foundation of the engineering discipline traditionally called radiative heat transfer. In general, the interactions considered in this discipline are nonwork because electromagnetic radiation flow carries both energy and entropy. Under very special limiting conditions, these interactions become heat. These limiting conditions are of interest in the exposition of thermodynamics because they provide a practical proof of existence of the heat interactions defined in Ref. 7, and discussed in a companion paper presented at this meeting [8]. In addition, the results provide a vivid illustration of entropy as a property of a system because even a single photon may have entropy in the same sense that it has energy.

2 Stable-Equilibrium-State Properties of the Electromagnetic Field

The stable-equilibrium-state principle implies that within the set of the stable equilibrium states of a system the value of any property is fully and uniquely determined by the values of the energy, the amounts of constituents, and the parameters such as the volume. In particular, for a system with volume V as the only parameter, and amounts of its r constituents denoted by $n = \{n_1, n_2, \ldots n_r\}$, the entropy S of the stable equilibrium states is fully and uniquely determined by the value of the energy, U, and the values of V and n so that

$$S = S(U, V, n) \tag{1}$$

We emphasize that, in our exposition [7], this conclusion is reached without any extraneous considerations such as lack of information, difficulty associated with complicated calculations, unpredictability of initial conditions, lack of interest in making detailed analyses of large systems, weak interactions between members of a statistical ensemble, or equilibrium with a reservoir. It is a precise, simple, exact, general, and far-reaching consequence of the first and second laws of thermodynamics about the existence of very restrictive interrelations among the properties of stable equilibrium states of any system.

For each system and most ranges of conditions, no explicit analytical expression of the fundamental relation (1) is available because the mathematical expressions of the interrelations between stable-equilibrium-state properties are transcendental. So the stable-equilibrium-state principle and its innumerable implications are used mostly to provide guidance about the number of properties that need be considered, about interpolations between and extrapolations of experimental results, and about procedures for carrying out measurements of properties. Exceptions to these general observations obtain for either special ranges of conditions, or special systems, or both. Examples of these exceptions are the high-temperature, low-pressure behavior of any substance (ideal gases), and the high temperature behavior of the electromagnetic field in a cavity.

For our purposes, we consider first the high temperature stable equilibrium states of the electromagnetic field in a cavity of volume V that confines all frequency modes. For this system, there is only one constituent, that is, the electromagnetic field. Its amount is fixed and equal to unity, that is, equation (1) becomes $S = S(U, V)$ because n is fixed and equal to unity. The fundamental relation is given by the expression

$$S = \frac{4}{3}(aVU^3)^{1/4} \tag{2}$$

where $a = 8\pi^5 k^4/15h^3c^3 = 7.565 \times 10^{-16}$ J/m^{3}K^4, k is the Boltzmann constant ($k = 1.38066 \times 10^{-23}$ J/K), h the Planck constant ($h = 6.6260 \times 10^{-34}$ Js), and c the speed of light in vacuum ($c = 2.9979 \times 10^8$ m/s).

Using equation (2) and the definitions of temperature. $T = 1/[(\partial S/\partial U)_V]$, and pressure, $p = T(\partial S/\partial V)_U$, we find the expressions for the internal energy $U = U(S, V)$, the temperature $T = T(S, V)$ (also $T = T(U, V)$), and the pressure $p = p(S, V)$ (also, $p = p(U, V)$):

$$U = \left(\frac{3}{4}\right)^{4/3}\left(\frac{S^4}{aV}\right)^{1/3} \tag{3}$$

$$T = \left(\frac{3S}{4aV}\right)^{1/3} = \left(\frac{U}{aV}\right)^{1/4} \tag{4}$$

$$p = \frac{1}{3}\left(\frac{3}{4}\right)^{4/3}\frac{1}{V}\left(\frac{S^4}{aV}\right)^{1/3} = \frac{U}{3V} \tag{5}$$

Rearranging these expressions, and defining the energy per unit volume $u = U/V$ and the entropy per unit volume $s = S/V$, we find

$$u = \frac{U}{V} = aT^4 \tag{6}$$

$$s = \frac{S}{V} = \frac{4}{3}aT^3 \tag{7}$$

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$$p = \frac{1}{3} a T^4 \tag{8}$$

Next, we consider the high temperature stable equilibrium states of the electromagnetic field in a cavity of volume V that confines only the modes with frequencies between ν and $\nu + d\nu$ in the limit as $d\nu$ tends to zero. The fundamental relation for this system can be expressed as

$$dS(\nu) = \frac{k\, dU(\nu)}{h\nu}\left[\left(1 + \frac{bV\nu^3 d\nu}{dU(\nu)}\right)\ln\left(1 + \frac{bV\nu^3 d\nu}{dU(\nu)}\right) - \frac{bV\nu^3 d\nu}{dU(\nu)}\ln\frac{bV\nu^3 d\nu}{dU(\nu)}\right] \tag{9}$$

where $b = 8\pi h/c^3 = 5.553 \times 10^{-57}$ Js^4/m^3.

Defining $U_\nu = dU(\nu)/d\nu$, $S_\nu = dS(\nu)/d\nu$,

$$u_\nu = \frac{U_\nu}{V} = \frac{1}{V}\frac{dU(\nu)}{d\nu} \tag{10}$$

$$s_\nu = \frac{S_\nu}{V} = \frac{1}{V}\frac{dS(\nu)}{d\nu} \tag{11}$$

and substituting in equation (9), we find

$$s_\nu = \frac{k u_\nu}{h\nu}\left[\left(1 + \frac{b\nu^3}{u_\nu}\right)\ln\left(1 + \frac{b\nu^3}{u_\nu}\right) - \frac{b\nu^3}{u_\nu}\ln\frac{b\nu^3}{u_\nu}\right] \tag{12}$$

Moreover, using the definition of temperature, we find

$$\frac{1}{T} = \left(\frac{\partial dS(\nu)}{\partial dU(\nu)}\right)_V = \frac{k}{h\nu}\ln\left(1 + \frac{b\nu^3}{u_\nu}\right) \tag{13}$$

$$u_\nu = \frac{8\pi h\nu^3/c^3}{\exp(h\nu/kT) - 1} \tag{14}$$

$$s_\nu = \frac{8\pi k\nu^2}{c^3}\left(\frac{h\nu/kT}{\exp(h\nu/kT) - 1} + \ln\frac{1}{1 - \exp(-h\nu/kT)}\right) \tag{15}$$

Of course, we can express the same results also in terms of the wavelength $\lambda = c/\nu$. For a cavity that confines the modes with frequencies between $\lambda = c/\nu$ and $\lambda + d\lambda$ the fundamental relation can be written as

$$dS(\lambda) = \frac{\lambda k\, dU(\lambda)}{hc}\left[\left(1 + \frac{b'V\, d\lambda}{\lambda^5 dU(\lambda)}\right)\ln\left(1 + \frac{b'V\, d\lambda}{\lambda^5 dU(\lambda)}\right) - \frac{b'V\, d\lambda}{\lambda^5 dU(\lambda)}\ln\frac{b'V\, d\lambda}{\lambda^5 dU(\lambda)}\right] \tag{16}$$

where $b' = 8\pi hc = 4.992 \times 10^{-24}$ Jm.

Defining $U_\lambda = dU(\lambda)/d\lambda, S_\lambda = dS(\lambda)/d\lambda$,

$$u_\lambda = \frac{U_\lambda}{V} = \frac{1}{V}\frac{dU(\lambda)}{d\lambda} \tag{17}$$

$$s_\lambda = \frac{s_\lambda}{V} = \frac{1}{V}\frac{dS(\lambda)}{d\lambda} \tag{18}$$

we find

$$s_\lambda = \frac{\lambda k u_\lambda}{hc}\left[\left(1 + \frac{b'}{\lambda^5 u_\lambda}\right)\ln\left(1 + \frac{b'}{\lambda^5 u_\lambda}\right) - \frac{b'}{\lambda^5 u_\lambda}\ln\frac{b'}{\lambda^5 u_\lambda}\right] \tag{19}$$

$$\frac{1}{T} = \left(\frac{\partial dS(\lambda)}{\partial dU(\lambda)}\right)_V = \frac{\lambda k}{hc}\ln\left(1 + \frac{b'}{\lambda^5 u_\lambda}\right) \tag{20}$$

$$u_\lambda = \frac{8\pi hc/\lambda^5}{\exp(hc/\lambda kT) - 1} \tag{21}$$

$$s_\lambda = \frac{8\pi k}{\lambda^4}\left(\frac{hc/\lambda kT}{\exp(hc/\lambda kT) - 1} + \ln\frac{1}{1 - \exp(-hc/\lambda kT)}\right) \tag{22}$$

A graph of the dimensionless spectral energy density $u_\nu/(8\pi h\nu^3/c^3)$ (or $u_\lambda/(8\pi hc/\lambda^5)$) versus dimensionless spectral entropy density $s_\nu/(8\pi k\nu^2/c^3)$ (or $s_\lambda/(8\pi k/\lambda^4)$) for a given frequency ν ($\lambda = c/\nu$) is shown in Figure 1.

Using the definition of pressure, we find

$$dp(\nu) = T\left(\frac{\partial dS(\nu)}{\partial V}\right)_{dU(\nu)} = \frac{kTb\nu^2 d\nu}{h}\ln\left(1 + \frac{u_\nu}{b\nu^3}\right) \tag{23}$$

$$dp(\lambda) = T\left(\frac{\partial dS(\lambda)}{\partial V}\right)_{dU(\lambda)} = \frac{kTb' d\lambda}{\lambda^4 hc}\ln\left(1 + \frac{\lambda^5 u_\lambda}{b'}\right) \tag{24}$$

Moreover, defining $p_\nu = dp(\nu)/d\nu$ and $p_\lambda = dp(\lambda)/d\lambda$, we find

$$p_\nu = \frac{8\pi kT\nu^2}{c^3}\ln\frac{1}{1 - \exp(-h\nu/kT)} \tag{25}$$

$$p_\lambda = \frac{8\pi kT}{\lambda^4}\ln\frac{1}{1 - \exp(-hc/\lambda kT)} \tag{26}$$

Finally, another important property of the field is the number of photons $dN(\nu)$—the quanta of the field—in the modes with frequencies between ν and $\nu + d\nu$, or the number of photons $dN(\lambda)$

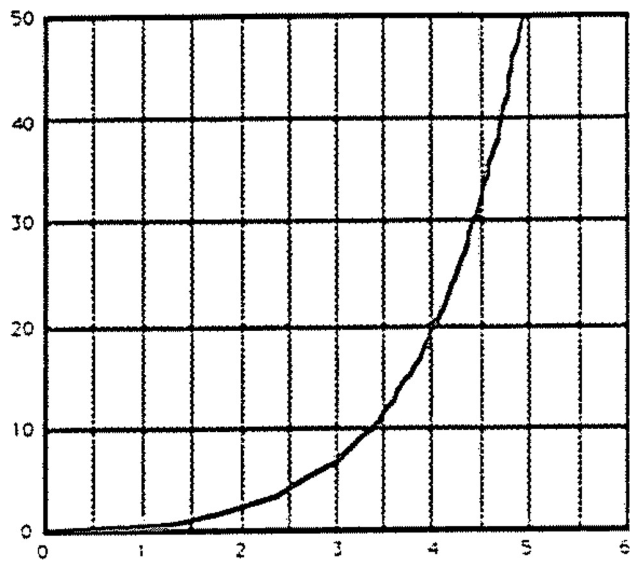

Fig. 1 Graph of dimensionless spectral energy density

$$y = \frac{u_\nu}{8\pi h\nu^3/c^3} = \frac{u_\lambda}{8\pi hc/\lambda^5}$$

versus dimensionless spectral entropy density

$$x = \frac{s_\nu}{8\pi h\nu^2/c^3} = \frac{s_\lambda}{8\pi k/\lambda^4}$$

for the stable equilibrium states of radiation modes with frequency ν and wavelength $\lambda = c/\nu$. The equation of the curve is

$$x = y\left[\left(1 + \frac{1}{y}\right)\ln\left(1 + \frac{1}{y}\right) - \frac{1}{y}\ln\frac{1}{y}\right]$$

with wavelengths between λ and $\lambda + d\lambda$. In a mode with frequency ν. each photon contributes an energy $h\nu$ (or hc/λ for $\lambda = c/\nu$). Thus, $dN(\nu) = dU(\nu)/h\nu, dN(\lambda) = dU(\lambda)\lambda/hc$ and, defining $n_\nu = dN(\nu)/Vd\nu$ and $n_\lambda = dN(\lambda)/Vd\lambda$, we have

$$n_\nu = \frac{1}{V}\frac{dN(\nu)}{d\nu} = \frac{u_\nu}{h\nu} = \frac{8\pi\nu^2/c^3}{\exp(h\nu/kT) - 1} \tag{27}$$

$$n_\lambda = \frac{1}{V}\frac{dN(\lambda)}{d\lambda} = \frac{u_\lambda}{hc/\lambda} = \frac{8\pi/\lambda^4}{\exp(hc/\lambda kT) - 1} \tag{28}$$

The expression $(8\pi\nu^2/c^3)d\nu$ is the number of modes with frequencies between ν and $\nu + d\nu$, and the expression $(8\pi/\lambda^4)d\lambda$ the number of modes with wavelengths between λ and $\lambda + d\lambda$.

It is noteworthy that the electromagnetic field in a cavity that confines all modes may be regarded as a composite of noninteracting, uncorrelated systems, each consisting of one of the modes. In other words, the field can be viewed as the superposition of the various modes that are confined in the cavity, each being entirely unaffected by the presence of other modes. This is a general feature of the electromagnetic field, valid for all states.

Here, we verify it for the stable equilibrium states by noting that

$$\int_0^\infty u_\nu(\nu, T)d\nu = \int_0^\infty u_\lambda(\lambda, T)d\lambda = u = aT^4 \tag{29}$$

$$\int_0^\infty s_\nu(\nu, T)d\nu = \int_0^\infty s_\lambda(\lambda, T)d\lambda = s = \frac{4}{3}aT^3 \tag{30}$$

$$\int_0^\infty p_\nu(\nu, T)d\nu = \int_0^\infty p_\lambda(\lambda, T)d\lambda = p = \frac{1}{3}aT^4 \tag{31}$$

$$\int_0^\infty n_\nu(\nu, T)d\nu = \int_0^\infty n_\lambda(\lambda, T)d\lambda = n = \frac{30\zeta(3)}{\pi^4}\frac{a}{k}T^3 \tag{32}$$

where $u_\nu(\nu, T), u_\lambda(\lambda, T), s_\nu(\nu, T), s_\lambda(\lambda, T), p_\nu(\nu, T)$, and $p_\lambda(\lambda, T)$ are given respectively by equations (14), (21), (15), (22), (25). and (26). and $\zeta(3) = 1.2020569$. the value of the Riemann zeta function, $\zeta(x) = \sum_{q=1}^\infty 1/q^x$ for $x = 3$

Comparison of equations (29) and (6) [and (31) and (8)] shows that the energy of the field with all the modes equals the sum of the energies of the modes, each confined by itself, and each at the same temperature as the field with all the modes. This additivity of energy proves that the various modes are separable, that is. noninteracting. Comparison of equation (30) with (7) shows that the entropy of the field with all the modes equals the sum of the entropies of the modes each confined by itself at the same temperature T. This additivity of entropy proves that the various modes are uncorrelated, that is, independent.

The fact that the various modes do not interact with each other means that modes cannot exchange energy and entropy with one another. Only interactions with matter, for example the atoms and molecules of the walls of the confining cavity, can promote indirectly exchanges between different modes. Another intriguing aspect is that each mode in a stable equilibrium state has both energy and entropy.

It is interesting to evaluate the photon compressibility ratio for the field with all modes, the field with only the modes between ν and $\nu + d\nu$, and the field with only the modes between λ and $\lambda + d\lambda$, that is. respectively,

$$\frac{p}{nkT} = \frac{\pi^4}{90\zeta(3)} = 0.90039 \tag{33}$$

$$\frac{p_\nu}{n_\nu kT} = [\exp(h\nu/kT) - 1]\ln\frac{1}{1 - \exp(-h\nu/kT)} \tag{34}$$

$$\frac{p_\lambda}{n_\lambda kT} = [\exp(hc/\lambda kT) - 1]\ln\frac{1}{1 - \exp(-hc/\lambda kT)} \tag{35}$$

For the low frequency modes, i.e., as $h\nu/kT = hc/\lambda kT \to 0$ the photon compressibility goes to zero, whereas for the high frequency modes, i.e., as $h\nu/kT = hc/\lambda kT \to \infty$. it goes to unity.

Graphs of the dimensionless spectral distributions of energy, entropy, pressure and photon density are shown in Figures 2 and 3.

The Gibbs free energy per unit volume of each mode of the field is zero, i.e.,

$$u_\nu - Ts_\nu + p_\nu = 0$$

and so is the Gibbs free energy of the field. By simple differentiation we can readily verify the interesting relations

$$\frac{du/dT}{ds/dT} = \frac{\partial u_\nu/\partial T}{\partial s_\nu/\partial T} = \frac{\partial u_\lambda/\partial T}{\partial s_\lambda/\partial T} = T \tag{37}$$

that we use in Section 5.

3 Energy and Entropy Flow Through Radiation

Among the properties of the electromagnetic field, velocity (i.e. speed and direction) of propagation plays an important role. Inside the cavity, that is, in the absence of interactions with matter, a measurement of the speed of propagation of radiation in any state always results in the value c, the speed of light in vacuum. In addition, for the field in a stable equilibrium state in an isotropic cavity, measurement results of velocity are uniformly distributed over the entire solid angle 4π, that is, the fraction of measurement results in directions that lie within a given solid angle $d\Omega$ is equal to $d\Omega/4\pi$. Thus, the value of the velocity of the field in a stable

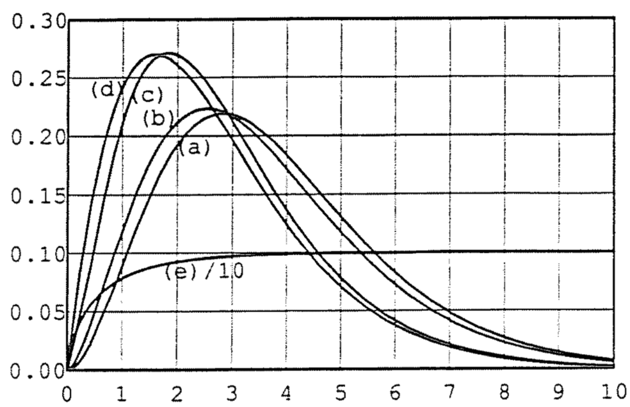

Fig. 2 Graphs as functions of $x = h\nu/kT$. Curve (e) is scaled by a factor of 1/10. The area under each if the curves (a), (b), (c), and (d) is unity. The maxima occur respectively at (a) $x = 2.82144$. (b) $x = 2.53823$. (c) $x = 1.83030$ and (d) $x = 1.59362$.

$$\frac{kT}{h}\frac{u_\nu}{u} = \frac{15}{\pi^4}\frac{x^3}{e^x - 1} \tag{a}$$

$$\frac{kT}{h}\frac{s_\nu}{s} = \frac{45}{4\pi^4}x^2\left(\frac{x}{e^x - 1} + \ln\frac{1}{1 - e^{-x}}\right) \tag{b}$$

$$\frac{kT}{h}\frac{p_\nu}{p} = \frac{45}{\pi^4}x^2\ln\frac{1}{1 - e^{-x}} \tag{c}$$

$$\frac{kT}{h}\frac{n_\nu}{n} = \frac{1}{2\zeta(3)}\frac{x^2}{e^x - 1} \tag{d}$$

$$\frac{p_\nu}{n_\nu kT} = (e^x - 1)\ln\frac{1}{1 - e^{-x}} \tag{e}$$

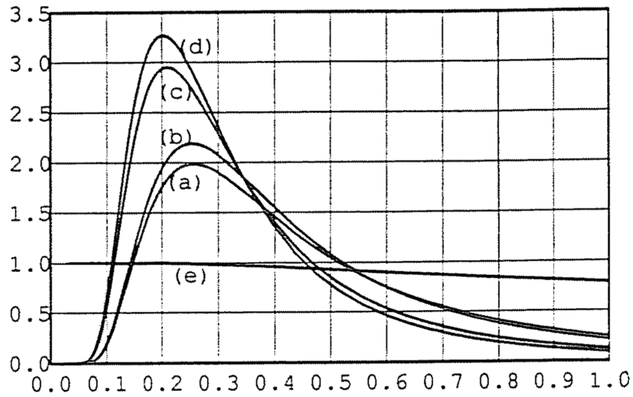

Fig. 3 Graphs as functions of $y = \lambda kT/hc$. The area under each of the curves (*a*), (*b*), (*c*), and (*d*) is unity. The maxima occur respectively at (*a*) $y = 0.201405$, (*b*) $y = 0.208713$. (*c*) $y = 0.252417$, and (*d*) $y = 0.255057$.

$$\frac{hc}{kT}\frac{u_\lambda}{u} = \frac{15}{\pi^4}\frac{1/y^5}{e^{1/y}-1} \tag{a}$$

$$\frac{hc}{kT}\frac{s_\lambda}{s} = \frac{45}{4\pi^4}\frac{1}{y^4}\left(\frac{1/y}{e^{1/y}-1} + \ln\frac{1}{1-e^{-1/y}}\right) \tag{b}$$

$$\frac{hc}{kT}\frac{p_\lambda}{p} = \frac{45}{\pi^4}\frac{1}{y^4}\ln\frac{1}{1-e^{-1/y}} \tag{c}$$

$$\frac{hc}{kT}\frac{n_\lambda}{n} = \frac{1}{2\zeta(3)}\frac{1/y^4}{e^{1/y}-1} \tag{d}$$

$$\frac{p_\lambda}{n_\lambda kT} = (e^{1/y}-1)\ln\frac{1}{1-e^{-1/y}} \tag{e}$$

equilibrium state, i.e, the mean value of all measurement results, is zero.

If it communicates with vacuum through a small aperture, the cavity behaves as a black body. Radiation propagates through the aperture out of the cavity with speed c and directions uniformly distributed over the outward solid angle 2π. At the aperture the radiation is no longer in a stable equilibrium state because it has a nonzero (mean) value of velocity. It is a nonequilibrium state. For a sufficiently short time interval, however, we assume that the presence of the small aperture does not perturb significantly the stable equilibrium state of the field inside the cavity, and that velocity measurement results in the neighborhood of the aperture are identical to the corresponding results inside the cavity for the outward directions, but are nil for the inward directions.

Accordingly, the fraction of measurements that result in directions that lie within a given solid angle $d\Omega$ is equal to $d\Omega/4\pi$ for the outward directions, and zero for the inward directions. To find the mean value $c_\perp$ of the velocity component normal to the aperture surface, we denote by θ the angle between the velocity direction and the outward normal to the aperture surface, and integrate over all outward directions, so that

$$c_\perp = \int c\cos\theta\frac{d\Omega}{4\pi} = \int_0^{2\pi} d\phi\int_0^{\pi/2} c\cos\theta\sin\theta\, d\theta = \frac{c}{4} \tag{38}$$

because $d\Omega = \sin\theta d\theta d\phi$.

The volume of radiation which in a time interval dt flows across the element of area da of the aperture surface is $c_\perp\, dt\, da$, that is, $c_\perp$ represents also the volume rate per unit area and unit time or volume flux of radiation crossing the aperture surface from inside to outside the cavity. Because inside the cavity each unit volume contains radiation with energy u, entropy s and photon number n, with the volume flux are associated also an energy flux $uc_\perp$, an entropy flux $sc_\perp$, and a photon flux $nc_\perp$, so that

$$J_u^{\rightarrow} = \frac{c}{4}u = \sigma T^4 \tag{39}$$

$$J_s^{\rightarrow} = \frac{c}{4}s = \frac{4}{3}\sigma T^3 \tag{40}$$

$$J_n^{\rightarrow} = \frac{c}{4}n = \frac{30\zeta(3)}{\pi^4 k}\sigma T^3 \tag{41}$$

where we use equations (6) to (8), the Stephan-Boltzmann constant $\sigma = ca/4 = 2\pi^5k^4/15h^3c^2 = 5.67083\times10^{-8}$ W/m^{2}K^4, and $30\zeta(3)\sigma/\pi^4k = 1.52057\times10^{15}$ l/m^{2}sK^3.

Similarly, with the volume flux are also associated the energy spectral flux density per unit frequency range $J_{u\nu}^{\rightarrow} = u_\nu c/4$ or per unit wavelength range $J_{u\lambda}^{\rightarrow} = u_\lambda c/4$, the entropy spectral flux density per unit frequency range $J_{s\nu}^{\rightarrow} = s_\nu c/4$ or per unit wavelength range $J_{s\lambda}^{\rightarrow} = s_\lambda c/4$, and the photon spectral flux density per unit frequency range $J_{n\nu}^{\rightarrow} = n_\nu c/4$ or per unit wavelength range $J_{n\nu}^{\rightarrow} = n_\lambda c/4$.

More generally, for an aperture that allows only outward directions within a cone of apex angle δ centered around the normal to the aperture surface, we find

$$c_\perp(\delta) = \int c\cos\theta\frac{d\Omega}{4\pi} = \int_0^{\delta} c\cos\theta\frac{2\pi\sin\theta d\theta}{4\pi} = \frac{c}{4}\sin^2\delta \tag{42}$$

For such an aperture we have

$$J_u^{\rightarrow}(\delta) = \frac{c}{4}u\sin^2\delta = \sigma T^4\sin^2\delta \tag{43}$$

$$J_s^{\rightarrow}(\delta) = \frac{c}{4}s\sin^2\delta = \frac{4}{3}\sigma T^3\sin^2\delta \tag{44}$$

$$J_n^{\rightarrow}(\delta) = \frac{c}{4}n\sin^2\delta = \frac{30\zeta(3)}{\pi^4 k}\sigma T^3\sin^2\delta \tag{45}$$

and similar relations for the spectral flux densities. Clearly, equations (38) and (41) concide with (42) to (45), respectively, for $\delta = \pi/2$.

4 Energy and Entropy Exchanges Through Radiation

Now we study the interaction between the electromagnetic fields in two cavities A and B that communicate with each other through a small aperture. We model the flow of radiation through the aperture by assuming that the field within each cavity is initially in a stable equilibrium state at temperatures T_A and T_B, respectively, and that for a sufficiently short time interval the presence of the small aperture does not perturb significantly either of the two stable equilibrium states. Of course, the state of the field in the vicinity of the aperture cannot be stable equilibrium, unless the fields in both cavities are at the same temperature. We assume that measurement results in directions that are outward for cavity A are identical to those that would obtain if cavity A communicates with vacuum, and measurement results in directions that are outward for cavity B are identical to those that would obtain if cavity B communicates with vacuum.

As a result of these assumptions, the fluxes at the aperture between the two cavities are nonzero, that is

$$J_\phi^{A\to B} = J_\phi^{A\rightarrow} - J_\phi^{B\rightarrow} = \frac{c}{4}(\phi_A - \phi_B) \tag{46}$$

where ϕ denotes any one of the volume densities u, s, n or spectral densities $u_\nu, s_\nu, n_\nu, u_\lambda, s_\lambda, n_\lambda, J^{A\rightarrow}$ is the flux (flux density per unit frequency range or wavelength range), out of an aperture between A and vacuum, and similarly $J^{B\rightarrow}$ is the flux from B to vacuum.

Specifically, for the energy, entropy, and photon fluxes in the direction from A to B normal to the aperture surface, we find

$$J_u^{A\rightarrow B} = \sigma(T_A^4 - T_B^4) \tag{47}$$

$$J_s^{A\rightarrow B} = \frac{4}{3}\sigma(T_A^3 - T_B^3) \tag{48}$$

$$J_n^{A\rightarrow B} = \frac{30\zeta(3)}{\pi^4 k}\sigma(T_A^3 - T_A^3) = \frac{1}{0.27766\,k}J_S^{A\rightarrow B} \tag{49}$$

Moreover, for the energy, entropy, and photon flux densities either per unit frequency range, or per unit wavelength range in the direction from A to B normal to the aperture surface, we find

$$J_{u_\nu}^{A\rightarrow B} = \frac{2\pi h\nu^3}{c^2}\left[\frac{1}{\exp(h\nu/kT_A)-1} - \frac{1}{\exp(h\nu/kT_B)-1}\right] \tag{50}$$

$$\begin{aligned} J_{s_\nu}^{A\rightarrow B} = \frac{2\pi k\nu^2}{c^2}\Bigg[&\frac{h\nu/kT_A}{\exp(h\nu/kT_A)-1} - \frac{h\nu/kT_B}{\exp(h\nu/kT_B)-1} \\ &+ \ln\frac{1-\exp(-h\nu/kT_B)}{1-\exp(-h\nu/kT_A)}\Bigg] \end{aligned} \tag{51}$$

$$\begin{aligned} J_{n_\nu}^{A\rightarrow B} &= \frac{2\pi\nu^2}{c^2}\left[\frac{1}{\exp(h\nu/kT_A)-1} - \frac{1}{\exp(h\nu/kT_B)-1}\right] \\ &= \frac{J_{u_\nu}^{A\rightarrow B}}{h\nu} \end{aligned} \tag{52}$$

$$J_{u_\lambda}^{A\rightarrow B} = \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{\exp(hc/\lambda kT_A)-1} - \frac{1}{\exp(hc/\lambda kT_B)-1}\right] \tag{53}$$

$$\begin{aligned} J_{s_\lambda}^{A\rightarrow B} = \frac{2\pi kc}{\lambda^2}\Bigg[&\frac{hc/\lambda kT_A}{\exp(hc/\lambda kT_A)-1} - \frac{hc/\lambda kT_B}{\exp(hc/\lambda kT_B)-1} \\ &+ \ln\frac{1-\exp(-hc/\lambda kT_B)}{1-\exp(-hc/\lambda kT_A)}\Bigg] \end{aligned} \tag{54}$$

$$\begin{aligned} J_{n_\lambda}^{A\rightarrow B} &= \frac{2\pi c}{\lambda^4}\left[\frac{1}{\exp(hc/\lambda kT_A)-1} - \frac{1}{\exp(hc/\lambda kT_B)-1}\right] \\ &= \frac{J_{u_\lambda}^{A\rightarrow B}}{hc/\lambda} \end{aligned} \tag{55}$$

Equations (50) to (55) can be used to evaluate the net exchange rates between two cavities that confine only modes within a given frequency or wavelength range, or between cavities confining all modes but communicating through a filtering aperture that is permeable only to modes within a given frequency or wavelength range.

5 Nonwork and Heat Interactions Through Radiation

We see clearly from equations (47) to (55) that the interaction between the fields in the two cavities A and B (through an all-passing or a filtering aperture) is an example of a non-work interaction. It is not a work interaction because it entails an exchange of entropy. In general, it is not a heat interaction because the temperature difference between the two interacting radiation fields is finite and not infinitesimal as required for a heat interaction [7,8], In particular, we note that the ratio of energy exchanged and entropy exchanged between the two cavities is well defined but different from the temperature of the radiation in either cavity, i.e,

$$\frac{J_u^{A\rightarrow B}}{J_s^{A\rightarrow B}} = \frac{u(T_A)-u(T_B)}{s(T_A)-s(T_B)} = \frac{3}{4}\frac{T_A^4 - T_B^4}{T_A^3 - T_B^3} \tag{56}$$

$$\frac{J_{u_\nu}^{A\rightarrow B}}{J_{s_\nu}^{A\rightarrow B}} = \frac{u_\nu(T_A)-u_\nu(T_B)}{s_\nu(T_A)-s_\nu(T_B)} \tag{57}$$

$$\frac{J_{u_\lambda}^{A\rightarrow B}}{J_{s_\lambda}^{A\rightarrow B}} = \frac{u_\lambda(T_A)-u_\lambda(T_B)}{s_\lambda(T_A)-s_\lambda(T_B)} \tag{58}$$

However, in the limit of an infinitesimal temperature difference between the fields in the two cavities, the interaction is heat, whether the aperture is all-passing or filtering, provided that both cavities confine either all modes or the same range of frequencies (wavelengths). We emphasize this limiting case because it constitutes an important experimentally verified proof of existence of heat interactions as defined in [7,8].

Indeed, for $T_A = T + dT, T_B = T$, and $dT \rightarrow 0$, we find

$$\frac{J_u^{A\rightarrow B}}{J_s^{A\rightarrow B}} = \frac{u(T+dT)-u(T)}{s(T+dT)-s(T)} = \frac{du/dT}{ds/dT} = T \tag{59}$$

$$\frac{J_{u_\nu}^{A\rightarrow B}}{J_{s_\nu}^{A\rightarrow B}} = \frac{u_\nu(T+dT)-u_\nu(T)}{s_\nu(T+dT)-s_\nu(T)} = \frac{\partial u_\nu/\partial T}{\partial s_\nu/\partial T} = T \tag{60}$$

$$\frac{J_{u_\lambda}^{A\rightarrow B}}{J_{s_\lambda}^{A\rightarrow B}} = \frac{u\lambda(T+dT)-u_\lambda(T)}{s_\lambda(T+dT)-s_\lambda(T)} = \frac{\partial u_\lambda/\partial T}{\partial s_\lambda/\partial T} = T \tag{61}$$

where in writing the last of each of equations (59) to (61) we use equations (37). We see that, in the limit of $T_A \rightarrow T_B = T$, each of the ratios of energy and entropy flows equals T as it should for a heat interaction.

Finally, we note that some authors [2,3] have defined a "temperature" even for states of radiation that are not stable equilibrium by the following procedure. The spectral energy density $u_\nu(\nu, \Omega)$ is measured, where Ω denotes direction. This density is used in equation (11), and a function $T(\nu, \Omega)$ is determined. In our view, calling $T(\nu, \Omega)$ a temperature is wrong and misleading. Temperature cannot be defined for states, such as nonequilibrium states, for which energy and entropy are not interdependent. Moreover, the function $T(\nu, \Omega)$ cannot be measured by a thermometer.

6 Conclusions

From a review of the properties of the stable equilibrium states of the electromagnetic field, we find that the interaction between two black bodies at different temperatures involves both energy and entropy flows. In general, this interaction is not heat because the ratio of the energy and entropy exchanged is not equal to the temperature of either black body. It is heat only in the limit of infinitesimal difference between the temperature of the two black bodies. This limit is an important experimental proof of existence of heat interactions as defined in Refs. 7 and 8.

It is noteworthy that the interaction between two black bodies A and B at different temperatures involves irreversibilities in both black bodies. The reason is that $J_u^{A\rightarrow B}/J_s^{A\rightarrow B} \neq (\partial U/\partial S)_{V^A} = T_A$ and $J_u^{B\rightarrow A}/J_s^{B\rightarrow A} \neq (\partial U/\partial S)_{V^B} = T_B$. In the limit of $T_A \rightarrow T_B$, however, both irreversibilities disappear as they should for heat interaction.

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