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A VARIABLE-CONDUCTANCE WALL BASED ON THE KNUDSEN CONDUCTION REGIME

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We describe the operation of a vessel with variable-conductance walls, based on the superposition of heat transfer by radiation and conduction through a rarefied saturated-vapor, in transition between the Knudsen and the continuum behavior. The rarefied vapor fills a narrow gap between two thin walls. We derive an analytical correlation to estimate the heat flux between the two thin walls and through the narrow gap filled by rarefied saturated-vapor. We discuss the heat transfer characteristics of such a container for some cases of technological interest. We present a set of design criteria.

NOMENCLATURE

| | |
|-------------|---|
| F_1, F_2 | dimensionless functions defined in Appendix A |
| F_{Kn} | function of the Knudsen number defined by Equation 10 |
| k | gas thermal conductivity |
| Kn | Knudsen number ($= \lambda / L$) |
| L | thickness of the gap |
| M_m | molar mass |
| p | gas pressure |
| q | heat flux by conduction and radiation defined by Equation 11 |
| q_k | heat flux by conduction defined by Equation 8 |
| q_k | heat flux by conduction in continuum regime defined by Equation 3 |
| q_λ | heat flux by conduction in Knudsen regime defined by Equation 6 |
| R_1 | curvature radius of the inside surface |
| R_2 | curvature radius of the outside surface |
| R_{th} | thermal resistance per unit area defined by Equation B.1 |

| | |
|---------------|------------------------------------|
| T | gas temperature |
| T_1 | temperature of the inside surface |
| T_2 | temperature of the outside surface |
| α | thermal accommodation coefficients |
| ε | surface emissivity |
| λ | molecular mean free path |
| σ | collision diameter |
| σ_0 | Stefan-Boltzman constant |

INTRODUCTION

In this paper we present a technical application of an effect based on the known phenomenology of heat transfer by conduction in a rarefied gas. The application allows us to devise a vessel with variable-conductance walls, i.e., a purely passive system that may keep its inside temperature nearly constant under a given thermal load at variable outside temperature.

It is well known that in a rarefied gas, when the molecular mean free path is much larger than the characteristic length of the container, the intermolecular interactions become weaker than the interactions between molecules and container walls and, therefore, the heat transfer effectiveness is strongly reduced.

Exploiting the pressure-temperature link for the equilibrium two-phase states, it is possible to change the degree of vapor-rarefaction and, therefore, the heat-transfer effectiveness, by a change of vapor temperature. Thus, with a suitable choice of substance and characteristic length of the container, we can obtain, as the inside temperature of the container varies in a relatively small range, a change of the mean free path of the vapor molecules from values much larger to values much smaller than the gap thickness and, consequently, a change of the heat transfer characteristics from an ineffective conduction regime (Knudsen-regime) to a more effective one (continuum-regime).

Based on this effect of transition between different conduction regimes, a variable-conductance wall can be devised: the wall is made of two thin walls divided by a narrow gap filled by a rarefied vapor in equilibrium with one of its condensed phase (liquid or solid). A container with such variable-conductance walls could limit the temperature range of electronic equipment that operates in an environment with severe temperature excursion.

In this paper we first derive an analytical correlation to estimate the heat flux between the two shells and through the narrow gap filled by rarefied saturated vapor. Then, we discuss the heat transfer characteristics of such a container for some cases of technological interest. Finally, we present a set of design criteria.

TRANSITION BETWEEN KNUDSEN REGIME AND CONTINUUM REGIME

We consider two thin walls divided by a narrow gap. The two walls may be plain, concentric cylindrical or concentric spherical. Let T_1 and T_2 be the temperatures respectively of the inside and outside surface, with $T_1 > T_2$, and R_1 and R_2 the curvature radii, respectively, of the inside and

outside surface, in the case of cylindrical or spherical walls. The gap between the two walls is filled by gas. Assuming a negligible convection intensity in the gas, the heat transfer between the walls is by conduction through the gas and by radiation between the surfaces. The heat flux on the inside surface, due to conduction only, is

$$q_k = \frac{F_1}{L} k (T_1 - T_2) \quad (1)$$

where F_1 is a dimensionless function, defined in Appendix A, which depends on geometry, L is the thickness of the gap, and k the thermal conductivity of gas.

For gases that may be considered to behave as ideal gases, i.e., for reduced pressure smaller than 0.1 or reduced temperature greater than 2, the thermal conductivity, as derived by elementary kinetic theory of gases (Reid et al., 1987), is

$$k = \frac{8.3224 \times 10^{-2}}{\sigma^2 \Omega_{22}} \sqrt{T/M_m} \quad (2)$$

where T is the gas temperature, M_m its molar mass, σ the collision diameter, Ω_{22} the principal collision integral; all variables are in SI units, except for σ that is expressed in Angstrom units. Substituting Equation 2 into Equation 1, this may be rewritten as

$$q_k = \frac{F_1}{L} \frac{8.3224 \times 10^{-2}}{\sigma^2 \Omega_{22}} \sqrt{T/M_m} (T_1 - T_2) \quad (3)$$

where for T we may assume the average value between T_1 and T_2 . Equation 2 and, therefore, Equation 3, hold under the further assumption that the molecular mean free path λ is much larger than the gap thickness L , i.e., for the Knudsen number, $Kn = \lambda/L$, much smaller than 1. For ideal gases, the Knudsen number becomes

$$Kn = \frac{1}{L} \frac{K}{\sqrt{2\pi\sigma^2}} \frac{T}{p} \quad (4)$$

since the mean free path, as derived by the kinetic theory (Warren and Willis, 1985), is

$$\lambda = \frac{K}{\sqrt{2\pi\sigma^2}} \frac{T}{p} \quad (5)$$

where K is the Boltzman constant and p the gas pressure.

When the molecular mean free path becomes large compared to the characteristic length, however, the gas does not behave any longer as a continuous medium and heat transfer becomes much smaller. In the free molecule or Knudsen regime ($Kn \gg 1$), the heat flux may be well approximated by Knudsen's formula (Kennard, 1938)

$$q_\lambda = F_2 (J/2 + 1) \mathfrak{R} \frac{P}{\sqrt{2\pi M_m \mathfrak{R} T}} (T_1 - T_2) \quad (6)$$

where F_2 is a dimensionless function defined in Appendix A, that depends on the surface characteristics, J is the number of degrees of freedom of the gas molecule, \mathfrak{R} is the gas constant. Since Equation 6 holds for small temperature differences only, i.e., $(T_1 - T_2)/T_2 \ll 1$, temperature T may be substituted by the average value between T_1 and T_2 . Substituting Equation 4 into Equation 6, this becomes

$$q_\lambda = \frac{F_2}{L} \frac{3.574 \times 10^{-4} (J/2 + 1)}{\sigma^2 \text{Kn}} \sqrt{T/M_m} (T_1 - T_2) \quad (7)$$

i.e., in Knudsen regime the heat flux q_λ is proportional to Kn^{-1} and, therefore, is vanishing for increasing Knudsen numbers. In addition, if Equation 7 is compared to Equation 6, we deduce that q_λ is much smaller than q_k .

Finally, when the mean free path is of the order of the characteristic length ($\text{Kn} \approx 1$), we have the transition-regime, and the heat flux has been evaluated by several different analytical methods as discussed by Springer (1971). For the general case, i.e., for an arbitrary value of the Knudsen number, the heat flux by conduction may be approximated by the simple interpolation formula

$$q_k = e^{-\text{Kn}} q_k + (1 - e^{-\text{Kn}}) q_\lambda \quad (8)$$

Equation 8 approximates correctly both the continuum limit ($\text{Kn} \ll 1$, $q_k \approx q_k$), and the free molecule limit ($\text{Kn} \gg 1$, $q_k \approx q_\lambda$), however, its validity in the transition region should be verified experimentally. For the purpose of the present simplified analysis, we adopt Equation 8 for simplicity, also in view of the lack of generally accepted correlation for the transition regime. Substituting Equations 3 and 7 into Equation 8, this becomes

$$q_k = \frac{1}{L} F_{\text{Kn}} \sqrt{T/M_m} (T_1 - T_2) \quad (9)$$

where F_{Kn} is a function of the Knudsen number defined as

$$F_{\text{Kn}} = \frac{8.3224 \times 10^{-2}}{\sigma^2 \Omega_{22}} F_1 e^{-\text{Kn}} + \frac{3.574 \times 10^{-4} (J/2 + 1)}{\sigma^2} F_2 \frac{1 - e^{-\text{Kn}}}{\text{Kn}} \quad (10)$$

If radiative heat transfer is also considered, the heat flux between the two walls is

$$q = \frac{1}{L} F_{\text{Kn}} \sqrt{T/M_m} (T_1 - T_2) + F_3 \sigma_0 (T_1^4 - T_2^4) \quad (11)$$

where F_3 is a dimensionless function defined in Appendix A, that depends on the surface characteristics, and σ_0 is the Stefan-Boltzman constant.

Equation 11 has two interesting limits. For $Kn > 10$, the conduction is characterized by the ineffective free molecule regime and, therefore, $q_k \approx q_\lambda \ll q_{rad}$; the heat flux becomes

$$q \approx q_{rad} = F_3 \sigma_0 (T_1^4 - T_2^4) \quad (11')$$

Whereas, for $Kn < 0.01$, the conduction is characterized by the continuum regime; thus, $F_{Kn} \approx F_{Kn}(0)$ and the heat flux becomes

$$q \approx q_{k-rad} = \frac{1}{L} F_{Kn}(0) \sqrt{T/M} (T_1 - T_2) + F_3 \sigma_0 (T_1^4 - T_2^4) \quad (11'')$$

The transition between these two limits (low conduction in the free molecule regime and high conduction in the continuum regime) may have an appealing technical application described in the following section, i.e., a variable-conductance wall.

A VARIABLE-CONDUCTANCE WALL

We consider a container with walls made of two shells divided by a narrow gap. We assume there is a power source inside the container so that its internal surface operates at fixed thermal flux. For simplicity, we assume that the outside surface operates, instead, at given uniform temperature. The internal temperature T_1 of container is a function of outside surface temperature, heat flux and wall thermal resistance, as defined in the Appendix B.

If the gap is evacuated, the heat transfer between the walls is due to radiation only. For a constant heat flux, the temperature T_1 is a strictly increasing function of T_2 . Instead, if the gap is filled by a gas, the heat transfer between the walls is due to radiation and conduction through the gas. However, for a given heat flux, the temperature T_1 has the same trend of the former case; in fact, when we choose the mass of gas, we fix the value of Knudsen number and, therefore, the value assumed by function F_{Kn} (Kn is a constant because it varies as T/p and thus as V/M that is a constant).

If instead the gap is filled by saturated vapor, i.e., vapor in equilibrium with its condensed phase (liquid or solid), the mass and, therefore, pressure of vapor are functions of its temperature, so that the Knudsen number (Equation 4) and F_{Kn} (Equation 10) are decreasing functions of temperature. Thus, for a constant heat flux, the temperature T_1 is no longer a strictly increasing function of T_2 but, within a certain range, may become a decreasing function of T_2 .

With a suitable choice of substance and geometric characteristics of the walls (gap thickness and surfaces emissivity), we may obtain a container with variable-conductance walls, i.e., a purely passive system that may keep its inside temperature nearly constant under a given thermal load at variable outside temperature.

Figure 1 shows the trend of temperature T_1 as function of T_2 for a heat flux of 50 W/m^2 , a gap thickness of 4.5 mm, and an emissivity of 0.4 for both surfaces. Curve (a) refers to the case of evacuated gap, i.e., the heat transfer is due to radiation only; curve (b), instead, refers to the case of gap filled with Xenon ($M_m = 131 \text{ g/mol}$) at a pressure of 1 bar for $T = 20^\circ \text{C}$ and, therefore, the heat

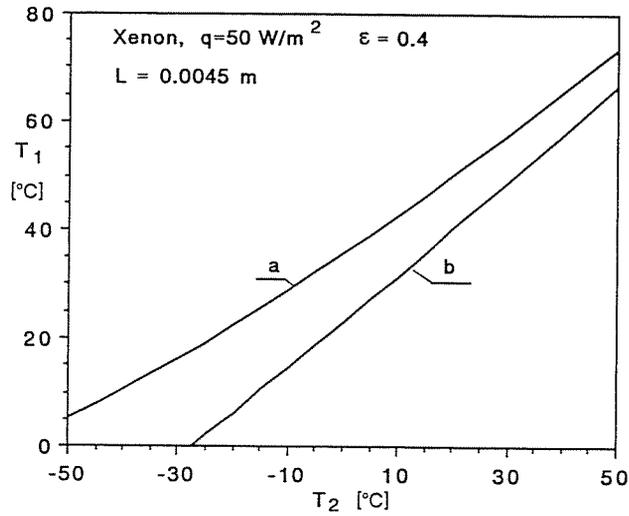


FIG. 1. Inside vs. outside temperature of the container walls with gap (a) evacuated, and (b) filled with Xenon.

transfer is by radiation between the surfaces and by conduction through the gas with a constant Knudsen ($Kn = 1.24 \times 10^{-5}$). For both cases, as already observed, the temperature T_1 is a strictly increasing function of T_2 ; obviously, for a given T_2 , temperature T_1 assumes a lower value in the latter case because of smaller thermal resistance of the wall.

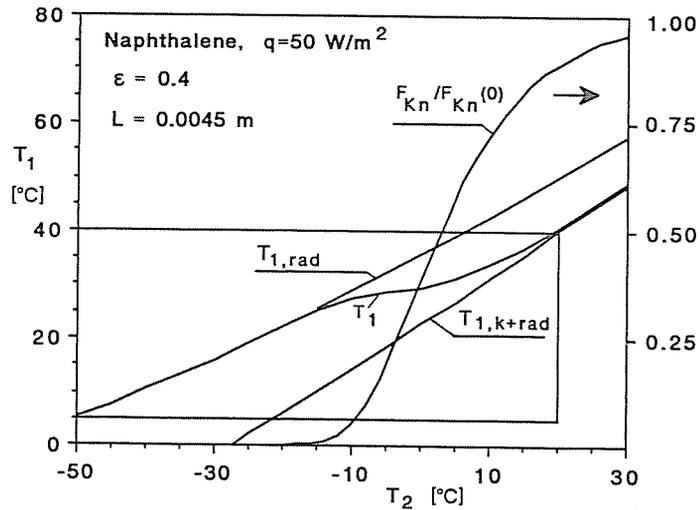


FIG. 2. Inside vs. outside temperature with gap filled by saturated-vapor of naphthalene.

Figure 2 shows the trend of temperature T_1 as function of T_2 when the gap, instead, is filled by saturated vapor of naphthalene ($M_m = 128 \text{ g/mol}$), for heat flux and wall geometric characteristics equal to the previous case. For low values of the outside temperature ($T_2 < -20 \text{ }^\circ\text{C}$), the pressure of naphthalene, equal to the saturation value corresponding to the temperature T_2 , is so low that the molecular mean free path is much larger than the gap thickness ($Kn \approx 11$, i.e., conduction is in the free-molecule regime). In this case, since the conductive thermal-resistance $R_{th,k}$ is much larger than the radiative one $R_{th,rad}$, the thermal resistance of the wall almost coincides with $R_{th,rad}$, and the temperature T_1 approximates $T_{1,rad}$, i.e., the temperature that the container achieves in the case of evacuated gap. Thus, the thermal-insulation effect of the wall has a maximum.

As the environment temperature increases, however, the sublimation pressure of naphthalene increases and the molecular mean free path decreases; consequently, the conductive and, therefore, the overall thermal resistance of the wall decrease. This reduction of the thermal-insulation effect of the wall, as the outside temperature increases, opposes the rise in inside temperature of the container.

Finally, for high values of outside temperature ($T_2 > 20 \text{ }^\circ\text{C}$), the pressure of naphthalene is relatively high and the molecular mean free path becomes small compared to the gap thickness ($Kn \approx 0.01$, i.e., conduction is in the continuum regime). The temperature T_1 approximates $T_{1,k+rad}$, i.e., the temperature that the container achieves if the gap is filled with gas at atmospheric pressure (continuum medium with thermal-conductivity k). In this case, the thermal-insulation effect of the wall has a minimum. Thus, whereas the outside temperature ranges between $-50 \text{ }^\circ\text{C}$ and $+20 \text{ }^\circ\text{C}$, the inside temperature of the container is kept weakly variable about the average value of $23 \text{ }^\circ\text{C}$ with an excursion of $10 \text{ }^\circ\text{C}$. Figure 2 also shows the trend of $F_{Kn}/F_{Kn}(0)$ as a function of the outside temperature. As can be seen, $F_{Kn}/F_{Kn}(0)$ has a value near to 0.5 at the flex point of the inside temperature curve.

Figure 3 shows the trend of temperature T_1 as a function of T_2 for a heat flux of 50 W/m^2 , a

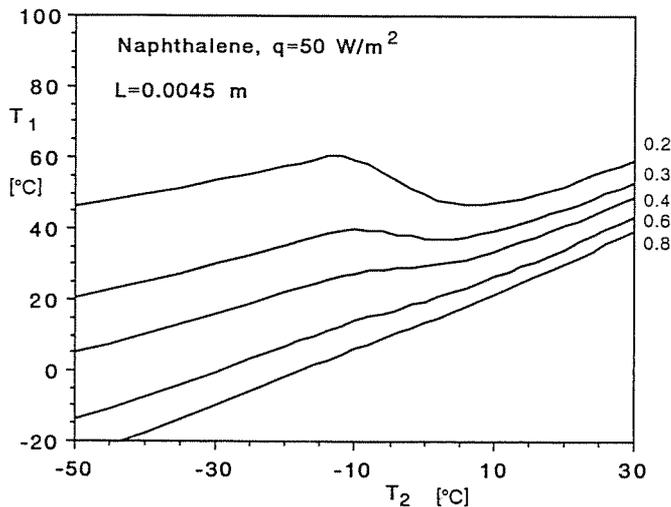


FIG. 3. Inside vs. outside temperature for different surface emissivities.

gap thickness of 4.5 mm, and emissivity, assumed equal for both surfaces, ranging between 0.2 and 0.8. As the emissivity increases, the radiative and overall thermal resistance decrease; consequently, the inside temperature of the container also decreases for a given outside temperature. In addition, for $\epsilon > 0.7$ the radiative resistance prevails on the conductive one, and temperature T_1 approximates $T_{1,\kappa+\text{rad}}$.

Finally, Figure 4 shows the trend of temperature T_1 as function of T_2 for a heat flux of 50 W/m^2 , an emissivity of 0.4 for both surfaces, and a gap thickness ranging between 0.5 and 15 mm. As the gap thickness decreases, the conductive thermal-resistance decreases but the Knudsen effect increases; consequently, the transition effect between conduction in the free-molecule regime and in the continuum regime is enhanced.

DESIGN CRITERIA

Now, we present simple criteria for the design of a container with variable-conductance walls. Specifically, for fixed ranges of environment temperature and internal temperature of the container, and for a given heat flux, we can determine the geometrical characteristics of the wall, i.e., gap thickness and surface emissivity, and some thermodynamic properties of the substance to be used.

First, for $T_2 = T_{2,\text{min}}$ we impose $T_1 = T_{1,\text{min}}$. Since thermal-insulation effect is maximum when the heat transfer is due to radiation only, the surface emissivity is

$$\epsilon = \frac{(1 + R^{*b}) q}{R^{*b} q + \sigma_0 (T_{1,\text{min}}^4 - T_{2,\text{min}}^4)} \quad (12)$$

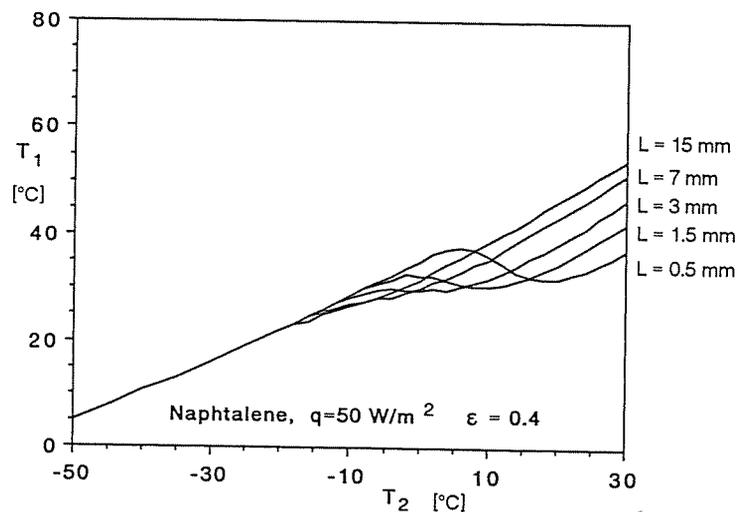


FIG. 4. Inside vs. outside temperature for different gap thickness.

provided that surfaces are characterized by the same emissivity, near-equal areas, and view-factor $F_{12} = 1$; otherwise it is

$$F_3 = \frac{q}{\sigma_0 (T_{1,\min}^4 - T_{2,\min}^4)} \quad (12')$$

Then, for $T_2 = T_{2,\max}$ we impose $T_1 = T_{1,\max}$. Since the thermal-insulation effect is minimum when the heat transfer is due to radiation and conduction in the continuum regime, the Knudsen number must be vanishing and, therefore, the gap thickness is

$$L = \frac{F_{Kn}(0)}{\sigma^2} \sqrt{\frac{T_{1,\max} + T_{2,\max}}{2M_m}} \frac{T_{1,\max} - T_{2,\max}}{q + F_3 \sigma_0 (T_{1,\min}^4 - T_{2,\min}^4)} \quad (13)$$

Since $F_{Kn} \approx F_{Kn}(0)$ for $Kn \leq 0.01$, from Equation 4 we can deduce the lowest value that the vapor pressure must achieve for $T_2 = T_{2,\max}$ and $T_1 = T_{1,\max}$

$$p_{\max} = p_{\text{sat}}(T_{2,\max}) \geq \frac{1}{L} \frac{K}{\sqrt{2\pi\sigma^2}} \frac{(T_{1,\max} + T_{2,\max})/2}{0.01} \quad (14')$$

where the vapor pressure has been assumed equal to the saturation value corresponding to $T_{2,\max}$, for the so-called "cold wall" principle.

Finally, since $F_{Kn} \approx 0.01 F_{Kn}(0)$ for $Kn \geq 10$, from Equation 4 we can also deduce the highest value that the vapor pressure must achieve for $T_2 = T_{2,\min}$ and $T_1 = T_{1,\min}$

$$p_{\min} = p_{\text{sat}}(T_{2,\min}) \leq \frac{1}{L} \frac{K}{\sqrt{2\pi\sigma^2}} \frac{(T_{1,\max} + T_{2,\max})/2}{10} \quad (14'')$$

Two values p_{\max} and p_{\min} of the saturation curve $p_{\text{sat}}(T)$ given by Equations 14' and 14'' can be used to choose the suitable substance to be used. In fact, for low values of vapor pressure we may assume that the saturation curve is well approximated by the simple formula

$$p = 10^{A - B/T} \quad (15')$$

Thus, substituting Equations 14 into Equation 15', we can evaluate the constants A and B

$$A = \frac{T_{2,\max} \text{Log } p_{\max} - T_{2,\min} \text{Log } p_{\min}}{T_{2,\max} - T_{2,\min}} \quad (15'')$$

$$B = (A - \text{Log } p_{\max}) T_{2,\max} \quad (15''')$$

SUMMARY AND CONCLUSIONS

We have described the operation of a vessel with variable-conductance walls, based on the superposition of heat transfer by radiation and conduction through a rarefied saturated-vapor, in transition between the Knudsen and the continuum behavior. The rarefied vapor fills a narrow gap between two thin walls. We have derived an analytical correlation to estimate the heat flux between the two thin walls and through the narrow gap filled by rarefied saturated-vapor. We have discussed the heat transfer characteristics of such a container for some cases of technological interest. We have presented a set of design criteria. Future works requires a better experimentally-based interpolation formula to describe the heat flux by conduction in a rarefied gas at the transition between the Knudsen and the continuum behavior. In addition, a numerical solution of the non-linear conduction problem with thermal conductivity depending on temperature should yield more accurate predictions.

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APPENDIX A

The function $F_1(R^*)$ is defined by the following equations

$$F_1(R^*) = 1 \tag{A.1'}$$

$$F_1(R^*) = \frac{R^* - 1}{R^* \ln R^*} \quad (\text{A.1}'')$$

$$F_1(R^*) = \frac{1}{R^*} \quad (\text{A.1}''')$$

that hold respectively for planar, cylindrical and spherical geometry; $R^* = R_1/R_2$ is the ratio between the curvature radii of the inside and outside walls. The function F_2 is defined as

$$F_2 = \left[\frac{1}{\alpha_1} + \frac{1 - \alpha_2}{\alpha_2} R^* b \right]^{-1} \quad (\text{A.2})$$

where α_1 and α_2 are the thermal accommodation coefficients, defined by Schaaf and Chambre (1958), respectively of the inside and outside surfaces; b is a constant equal to $\{0, 1, 2\}$ according to a planar, cylindrical or spherical geometry. For $\alpha_1 = \alpha_2 = \alpha$, Equation A.2 becomes

$$F_2(\alpha, R^*) = \frac{\alpha}{1 + (1 - \alpha) R^* b} \quad (\text{A.2}')$$

The function F_3 is defined as

$$F_3 = \frac{1}{A_1} \left[\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{F_{12} A_1} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2} \right]^{-1} \quad (\text{A.3})$$

where ε_1 and ε_2 are the emissivities respectively of the inside and outside surfaces, provided they can be modeled as gray surfaces, A_1 and A_2 are the areas of the inside and outside surfaces, and F_{12} is the view factor. For $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $F_{12} = 1$ Equation A.3 becomes

$$F_3(\varepsilon, R^*) = \frac{\varepsilon}{1 + (1 - \varepsilon) R^* b} \quad (\text{A.3}')$$

APPENDIX B

The thermal resistance of the wall per unit area is

$$R_{th} = \frac{T_1 - T_2}{q} \quad (\text{B.1})$$

where q is the heat flux given from Equation 11. The thermal resistance of the wall, per unit area, for purely conductive and purely radiative heat transfer, respectively, are

$$R_{th,k} = \frac{T_1 - T_2}{q_k} = \frac{L}{F_{Kn} \sqrt{T/M_m}} \quad (\text{B.2})$$

$$R_{th,rad} = \frac{T_1 - T_2}{q_{rad}} = \frac{1}{F_3 \sigma_0 (T_1^2 + T_2^2)(T_1 + T_2)} \quad (\text{B.3})$$

where q_k and q_{rad} are given from Equation 9 and 11', respectively. As $q = q_k + q_{rad}$, substituting Equations B.2 and B.3 into Equation B.1, this becomes

$$R_{th} = \left[R_{th,k}^{-1} + R_{th,rad}^{-1} \right]^{-1} \quad (\text{B.4})$$

For $R_{th,k} \gg R_{th,rad}$, Equation B.4 yields $R_{th} \approx R_{th,rad}$.