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Optimal Power Production Scheduling in a Complex Cogeneration System with Heat Storage

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Abstract

We discuss the problem of finding an optimal electricity and heat production schedule subject to the constraints imposed by the need to match simultaneously both the district heating and the electric power loads. In particular, we refer to the relatively complex cogeneration and district heating system of the city of Brescia, Italy and present:

- a model for the definition of the dynamic programming problem of search of the optimal electricity and heat production schedule for the next 24 hours;
- a method for the solution of the optimal cogeneration scheduling problem, including load subdivision among the various turbines and burners of the system (this algorithm was implemented in a software).

The method, results and software are of course specific to the cogeneration system of the city of Brescia. However, several aspects of this work can be readily extended to other systems by modifying only the specific details concerning the simplified performance model based on actual data for each cogeneration unit, the constraints on the allowed ranges of operation, and the interconnections between units.

1 Introduction

The city of Brescia has a relatively complex cogeneration and district heating system; the oldest, largest and most advanced in Italy. In 1998, the district heating network served 31 Mm^3 of heated space (almost 85% of the overall volume of buildings in the city) with 1100 GWh of thermal energy. The cogeneration system is composed of a subsystem (CL) consisting of three multifuel burners, producing superheated steam for three independent backpressure turbines, and a hot water reservoir; in addition there is a waste-to-energy facility (TU) with efficient energy recovery (the largest and most environmentally advanced in Italy) which provides steam to a turbine that can operate both in condensation and in various mixed cogeneration configurations.

The purpose of this paper is to show how the problem of production scheduling can be approached in a relatively complex cogeneration and district heating system. The tools developed in this work provide both a way of distributing the production load on an hourly basis and a way to partition the load between different production units.

The main feature of a cogeneration system is that it is meant to satisfy two separate needs or demands (heat and electricity) by a simultaneous production of the two forms of energy. Since the daily demand curves for electricity and heat are commonly distinct and seldom concomitant, a form of storage of one of the two products is necessary in order to succeed in providing both products when requested. In the district heating plant of Brescia, it is possible to store the heat produced in excess during some hours of the day in the form of hot water and use it later on, when needed. There are, of course, several ways of subdividing the load by using a hot water reservoir, nevertheless the objective should be to choose the way that maximizes profit. Profit can be increased by either augmenting electricity production when it is more valuable, or trying to take advantage of those loads that are particularly cheap, or both. In addition, for a given rate of heat production and given feed and return temperatures, costs should be continuously minimized by choosing the best combination of power levels for the three boilers and turbines of subsystem CL, the best fuel composition, and the best configuration of the waste-to-energy facility TU, subject to a variety of constraints, that vary on a daily basis, and taking into account start up and shut down additional costs.

Figure 1 shows a schematic layout of the plant.

The cogeneration system is composed of two subsystems: CL and TU. CL consists of three multifuel burners (named, respectively, B1, B2 and B3) producing superheated steam (510°C, 100 bar) for three independent backpressure turbines (T1, T2 and T3) with vapor condensation in the district heating exchanger. Table 1 shows the electrical and thermal ratings of the three turbines. Burners B1 and B2 use methane and fuel oil, while

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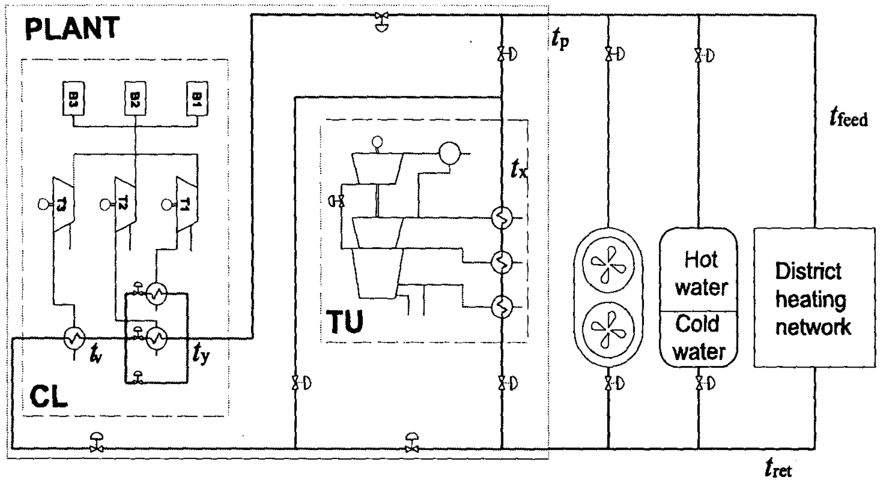

Fig. 1: Plant layout.

	T1	T2	T3
thermal power (MW)	84	87	130
electrical power (MW)	29	32	72

Table 1: Turbines.

B3 can also burn coal.

The waste-to-energy facility (TU) provides steam (450°C, 60 bar) to a turbine that can operate both in condensation (electrical power, 51 MW) and in various mixed cogeneration configurations (electrical power between 40 and 50 MW, thermal power about 110 MW). The TU subsystem can be connected either in series or in parallel to the CL subsystem.

The heat produced in excess can be either dissipated, by means of a cooling tower, or stored in a 2200 m^3 hot water reservoir.

2 Modeling the optimization problem

The optimization problem can be described as a multistage decision process, that is a process in which decisions are taken in sequence in order to obtain a specific result. Each decision implies a change in the state of the reservoir and, therefore, affects the results obtainable by the subsequent decisions.

The time span of the problem is subdivided in one-hour time intervals. For each time interval, decisions must be taken to choose the optimal values of the following variables that, for brevity, we call decisions:

- Q_{p}, overall amount of heat to be produced by TU and CL during the next hour;
- t_{p}, water temperature to be maintained at the exit of the plant during the next hour;
- $conf_{\mathrm{TU}}$, chosen configuration for the waste-to-energy facility TU for the next hour;
- $conf_{\mathrm{CL}} = (f\%, t_{\mathrm{v}})$, chosen configuration for the subsystem CL for the next hour, where $f\%$ is the fraction of water flow from the T3 condenser directed to the T2 condenser for the next hour and t_{v} is the water temperature at the exit of the T3 condenser and entering the T2 and T1 condensers;
- τ, subdivision of the load among boilers B2 and B3;
- σ_{i}, ρ, variables describing the choice of fuels to burn (methane, fuel oil, coal).

The choice of some of these variables, such as (Q_{p}, t_{p}), will affect the state of the reservoir, and, therefore, future states, whereas other variables affect only the momentary performance of the plant. The former variables are called *dynamic variables* and the latter *static variables.* Correspondingly, the optimization problem splits into a *dynamic problem* and a *static problem*, and the decision variables can be grouped as follows:

- Dynamic variables, $x_{\mathrm{d}} = (Q_{\mathrm{p}}, t_{\mathrm{p}})$;
- Static variables, $x_{\mathrm{s}} = (conf_{\mathrm{TU}}, conf_{\mathrm{CL}}, \tau, \sigma_{\mathrm{i}}, \rho)$.

2.1 The dynamic problem

The dynamic problem is defined in terms of stages, states, feasible decisions, transition functions, initial and final states, and objective function.

Stages. The optimal schedule is made of 24 decisions, each corresponding to the production in a specific hour of the day. Therefore the decision process consists of $N = 24$ stages.

States. At the beginning of each stage n, the reservoir will assume a specific state, s_n. The upper part of the reservoir will contain hot water with a temperature t_{res} between 90°C and 120°C. The remaining part contains cold water at t_{ret} (typically 60°C). The state is defined in terms of:

- $\bar{M}_{\text{res}}$ amount of hot water stored in the reservoir;
- t_{res} temperature of the hot water present in the reservoir.

The set S, of the admissible states s is defined as follows:

$$S = \{t_{\text{res}}, \bar{M}_{\text{res}} : \quad 90°\text{C} \le t_{\text{res}} \le 120°\text{C}, \quad 0 \le \bar{M}_{\text{res}} \le 2200\,\text{m}^3\}$$

Feasible decisions. At each stage, for every admissible state, we can define the set of feasible decisions, that is all the decisions that can be taken in those particular conditions and that lead to an admissible state at the next stage. The set has the following constraints:

1. Q_{p} has an upper limit corresponding to the maximum plant capacity;

2. t_{p} must be such as to maintain the temperature t_{feed} of the water fed to the distinct heating network above the following contractual limit

$$t_{\text{feed}} \ge \begin{cases} 120 & \text{if } t_{\text{env}} \le -7°\text{C} \\ 90 & \text{if } t_{\text{env}} \ge 20°\text{C} \\ 120 - \frac{30}{27}(7 + t_{\text{env}}) & \text{otherwise} \end{cases} \tag{1}$$

   where t_{env} is the environmental temperature;

3. Q_{p} and t_{p} must be such as to request heat from the storage tank at a rate less than the maximum allowed.

Transition functions. The transition functions $(s_{n+1} = T_n(s_n, x_d))$ indicate the state s_{n+1} at the beginning of stage $n+1$ resulting from the choice of decision x_d at stage n, given the state s_n at stage n. The transition function varies depending on whether the reservoir is being charged

$$\bar{M}_{\text{res,n+1}} = \bar{M}_{\text{res,n}} + \frac{Q_{\text{p}} - Q_{\text{r}}(n)}{c\,(t_{\text{p}} - t_{\text{ret}})}\Delta t \tag{2}$$

$$t_{\text{res,n+1}} = t_{\text{ret}} + \frac{\bar{M}_{\text{res,n}}(t_{\text{res,n}} - t_{\text{ret}}) + (Q_{\text{p}} - Q_{\text{r}}(n))\Delta t/c}{\bar{M}_{\text{res,n}} + \frac{Q_{\text{p}} - Q_{\text{r}}(n)}{c\,(t_{\text{p}} - t_{\text{ret}})}\Delta t} \tag{3}$$

discharged

$$\bar{M}_{\text{res,n+1}} = \bar{M}_{\text{res,n}} + \frac{Q_{\text{p}} - Q_{\text{r}}(n)}{c\,(t_{\text{res,n}} - t_{\text{ret}})}\Delta t \quad \text{and} \quad t_{\text{res,n+1}} = t_{\text{res,n}} \tag{4}$$

or not altered

$$\bar{M}_{\text{res,n+1}} = \bar{M}_{\text{res,n}} \quad \text{and} \quad t_{\text{res,n+1}} = t_{\text{res,n}} \tag{5}$$

where $\Delta t = 1$ hour,

- $Q_{\text{r}}(n)$ is the amount of heat requested by the district heating network at a given hour of the day;
- t_{ret} is the temperature of the water returning to the plant from the distinct heating network, which remains approximately constant for each given day.

Initial and final states. The initial state is the state of the reservoir at the beginning of the 24-hour optimization time interval, e.g. at midnight. The final state must be the same as the initial, in order to consider each day as a different and separate optimization interval from the others. If the final state were different from the initial, production during one day would affect the results obtainable in the following days, that is, the optimization time interval would have to be longer then 24 hours.

Objective function. Our scope is to maximize profit from each one-day production, therefore we consider those economic variables that are related to decisions that affect profit. We shall not consider maintenance costs and amortization costs, because they are in no way affected by the type of decisions we are taking. On the other hand, we consider the returns that derive from the sale of electricity and heat, and the fuel costs. The objective function is therefore

$$\begin{aligned}(OF)_{\text{d}} = max \sum_{\text{n}=1}^{24} [& p_{\text{E}}(n)\,W_{\text{E}}(n) + p_{\text{Q}}(n)\,Q_{\text{p}}(n) \\ & -p_{\text{CH}_4}\,N_{\text{CH}_4}(n) - p_{\text{FO}}\,M_{FO}(n) \\ & -p_{\text{coal}}\,M_{coal}(n)]\end{aligned} \tag{6}$$

where $p_{\text{E}}(n)$ and $p_{\text{Q}}(n)$ are the unit prices of electricity and heat at stage n (the n-th hour of the day) and p_{CH_4}, p_{FO} and $p_{\text{coal}}$the unit costs of methane, fuel oil and coal and $W_{\text{E}}(n)$, $Q_{\text{r}}(n)$, $N_{\text{CH}_4}(n)$, $M_{\text{FO}}(n)$ and $M_{\text{coal}}(n)$ denote the electricity and heat production and the consumption of each type of fuel at stage n.

2.2 The static problem

The static problem maximizes hourly profit, choosing the best possible values for the static variables, for given values of the dynamic variables.

The static problem is solved by assuming given values for the external plant variables (Q_{p}, t_{p}, t_{ret}) and finding the best values for $conf_{\mathrm{TU}}$, $conf_{\mathrm{CL}}$, σ_{i}, ρ and τ.

In order to complete the definition of the optimization problem an accurate mathematical description of the plant interconnections and component performance is necessary, which consists of a simplified model of each turbine and burner of the system and of the waste-to-energy facility based on correlations obtained by means of a structured statistical analysis of the available real performance data.

Also the static problem can be cast as a multistage decision process and therefore formulated in terms of stages ($\hat{n}$), states ($\hat{s}_{\hat{n}}$) and transition functions, and solved using the methods of dynamic programming theory.

The static problem is made of $\hat{N} = 4$ stages (Figure 2). The first stage consists of choosing the more external variable ($conf_{\mathrm{TU}}$), given the external parameters describing the production of the plant ($\hat{s}_1$). The choice of the configuration for the TU subsystem determines the state ($\hat{s}_2$) in which the CL subsystem works, since it defines how much water enters CL and its temperature. The choice of $conf_{\mathrm{TU}}$ implies a certain electricity production W_{E}, so that values of $conf_{\mathrm{TU}}$ that yield inadequate electrical production ($W_{\mathrm{E}} < W_{\mathrm{E,min}}$) should not be accepted. The second stage consists of choosing how the loads must be subdivided between the three turbines belonging to CL ($conf_{\mathrm{CL}}$). As a consequence of this choice, each turbine requires a certain amount of steam to work as requested ($\hat{s}_3$). The load is therefore to be subdivided among the boilers, by choosing the variable τ, which results in state $\hat{s}_4$ (load for each burner). The last stage consists of choosing for the given load of each boiler, the best type and composition of fuel needed (ρ, σ_{i}), so that the overall costs of production can be determined.

The transition functions from one state to the other consist either of the relevant correlations obtained from statistical analyses of the actual performance data of each turbine group or of the equations resulting from the description of the interconnections. For instance, the transition function from $\hat{s}_3$ to $\hat{s}_4$, i.e., $\hat{s}_4 = \hat{T}_3(\hat{s}_3, x_{s,3})$, is

$$m_{1B} = m_1 \quad \text{and} \quad m_{2B} = \tau(m_2 + m_3) \quad \text{and} \quad m_{3B} = (1-\tau)(m_2 + m_3) \qquad (7)$$

The objective function of the static problem is

$$(OF)_{\mathrm{s}} = max(p_{\mathrm{E}}\, W_{\mathrm{E}} + p_{\mathrm{Q}}\, Q_{\mathrm{P}} - p_{\mathrm{CH_4}}\, N_{\mathrm{CH_4}} - p_{\mathrm{FO}}\, M_{\mathrm{FO}} - p_{\mathrm{coal}}\, M_{coal}) \qquad (8)$$

where p_{E} and p_{Q} are the prices that prevail at the stage of the dynamic problem at which the static problem is solved, and the other terms are as already defined. The value $(OF)_{\mathrm{s}}$ is the optimal hourly marginal profit that we also denote MP for brevity.

3 Solving the optimization problem

Both the static and the dynamic problems defined in Section 2 have been cast as dynamic programming problems and can be solved by virtue of the principle of optimality, first stated by Richard Bellman [2]:

> An optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

A direct consequence of this principle is that an optimal solution can be associated with each state of each stage regardless of the decisions that might have been previously taken, as long as they result in the same state at the same stage. Thus, given a state s, at stage n and two ways of reaching that state from the first stage, the optimal solution, i.e., the choices that must be taken at stage n and the following stages, is independent of which way has been chosen from the first stage to stage n.

The algorithm that implements this principle proceeds backwards and can be used to solve both the dynamic problem (s_{n}, T_{n}, $N = 24$) and the static problem ($\hat{s}_{\hat{\mathrm{n}}}$, $\hat{T}_{\hat{\mathrm{n}}}$, $\hat{N} = 4$).

The algorithm begins by associating an optimal solution with each state at the beginning of the last stage (stage N or $\hat{N}$). In this case the problem can be easily solved by simple enumeration since it consists of an optimization problem in terms of a few variables, namely the choice of x_N at stage N (i.e., $x_{\mathrm{d,N}}$ or $x_{\mathrm{s},\hat{\mathrm{N}}}$) . The solution of the optimization problem at stage N produces a function $f_{\mathrm{N}}(s)$ that associates with each state s the optimal solution value and a function $\delta_{\mathrm{N}}(s)$ that associates with the state an optimal policy (best decision, x^*_N, that can be taken at state s, stage N). $f_{\mathrm{N}}(s)$ will be the optimal value of the objective function restricted to the last stage (i.e., not considering the contribution of decisions at stages $n < N$).

Next, the decisions at stage $N-1$ can be taken into consideration. For each state we consider the different feasible decisions, each one of them leads to a particular state of the final stage through the

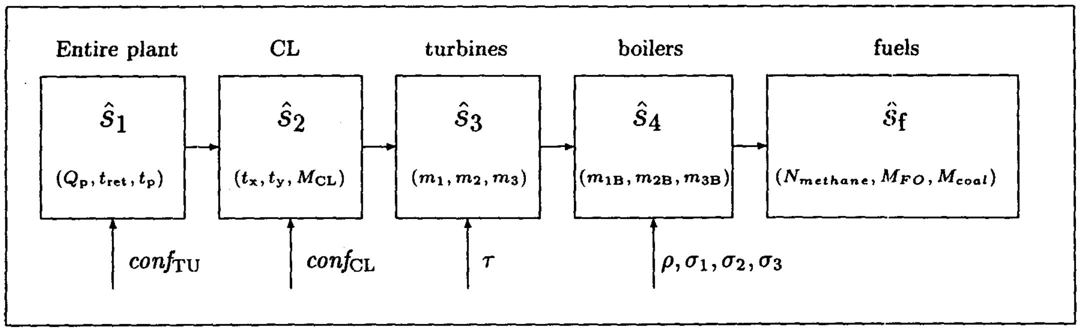

Fig. 2: Dynamic programming formulation of the static problem.

transition function. We need not reconsider what the best choice from there on is, since this is already known, that is $\delta_N(s)$. We just have to consider the additional contribution of the choices at the stage $N-1$. Also this optimization problem of just a few variables can be solved by simple enumeration and yields the function $f_{N-1}(s)$. The solution proceeds backwards for the other stages, as can be easily inferred by induction.

For the dynamic problem (Section 2.1), $f_n(s)$ has the following form

$$f_n(s) = max \sum_{i=n}^{24} [p_E(i)\, W_E(i) + p_Q(i)\, Q_p(i) - p_{CH_4}\, N_{CH_4}(i) - p_{FO}\, M_{FO}(i) - p_{coal}\, M_{coal}(i)]$$

which reduces to the whole-day objective function $(OF)_d$ (Equation 6) when $n = 1$ and s is the initial state $s_1 = (t_{res,1}, M_{res,1})$.

Once we reach the initial stage, we can determine the best path from any given state s_1 at stage 1 as follows

$$\begin{aligned} x_1 &= \delta_1(s_1) \\ s_2 &= T_1(s_1, x_1) \\ x_2 &= \delta_2(s_2) \\ &\dots \\ x_N &= \delta_N(s_N) \end{aligned} \qquad (9)$$

For the static problem (Section 2.2), $f_{\hat{n}}$ has a different form for each of the different stages.

For the last stage $\hat{N}$, $f_{\hat{N}}(\hat{s}_4)$ results from the minimization of the production costs for the boilers, given a specific value of the steam flow that must be produced by each one of them. We define

$$\begin{aligned} c(m_{1B}, m_{2B}, m_{3B}, \sigma_1, \sigma_2, \sigma_3, \rho) &= c(\hat{s}_4, x_{s,4}) \\ = p_{CH_4}\, N_{CH_4}(\hat{s}_4, x_{s,4}) &+ p_{FO}\, M_{FO}(\hat{s}_4, x_{s,4}) \\ &+ p_{coal}\, M_{coal}(\hat{s}_4, x_{s,4}) \end{aligned} \qquad (10)$$

as the function that describes the relation between production costs related to fuel consumption and the amount of steam production $\hat{s}_4 = (m_{1B}, m_{2B}, m_{3B})$ and the choice of fuels $x_{s,4} = (\sigma_1, \sigma_2, \sigma_3, \rho)$. Thus the objective function restricted to the last stage is

$$\begin{aligned} f_{\hat{N}}(\hat{s}_4) &= min_{\sigma_1, \sigma_2, \sigma_3, \rho}\, c(\hat{s}_4, \sigma_1, \sigma_2, \sigma_3, \rho) \\ &= min_{x_{s,4}}\, c(\hat{s}_4, x_{s,4}) \end{aligned}$$

Function 10 can be decomposed in terms of the costs related to each single boiler as follows

$$\begin{aligned} &c(m_{1B}, m_{2B}, m_{3B}, \sigma_1, \sigma_2, \sigma_3, \rho) \\ &= c_1(m_{1B}, \sigma_1) + c_2(m_{2B}, \sigma_2) + c_3(m_{3B}, \sigma_3, \rho) \end{aligned}$$

and $f_{\hat{N}}(\hat{s}_4)$ becomes

$$\begin{aligned} f_{\hat{N}}(\hat{s}_4) = min_{\sigma_1}\, c_1(m_{1B}, \sigma_1) &+ min_{\sigma_2}\, c_2(m_{1B}, \sigma_2) \\ &+ min_{\sigma_3, \rho}\, c_1(m_{3B}, \sigma_3, \rho) \end{aligned}$$

Therefore the fuel choice can be taken separately for each boiler and the optimization problem at stage $\hat{N}$ splits into three simpler problems, each associated with a single boiler. The optimal fuel choice for each boiler Bi for a given steam production m_{iB} determines the function $\delta_{4,i}(m_{iB})$. The overall δ_4 for this stage is

$$\begin{aligned} &\delta_4(m_{1B}, m_{2B}, m_{3B}) \\ &= (\delta_{4,1}(m_{1B}), \delta_{4,2}(m_{2B}), \delta_{4,3}(m_{3B})) \end{aligned}$$

At Stage 3 we decide how the production of steam requested by turbines T2 and T3 can be subdivided among boilers B2 and B3 in order to minimize joined production costs for B2 and B3. Therefore the objective function for this stage is

$$\begin{aligned} &f_3(m_1, m_2, m_3) \\ &= min_\tau\, c(m_{1B}, m_{2B}, m_{3B}, \delta_4(m_{1B}, m_{2B}, m_{3B})) \\ &= c_1(m_{1B}, \delta_{4,1}) + \\ &min_\tau \left(c_2(m_{2B}, \delta_{4,2}(m_{2B})) + c_3(m_{3B}, \delta_{4,3}(m_{3B}))\right) \\ &= min_\tau f_4(m_{1B}, m_{1B}, m_{1B}) \end{aligned}$$

subject to the transition function for stage 3

$$\begin{aligned} &(m_{1B}, m_{2B}, m_{3B}) = T_3(m_1, m_2, m_3, \tau) \\ &= (m_1, \tau(m_2 + m_3), (1-\tau)(m_2 + m_3)) \end{aligned}$$

The approach we are using allows us not to reconsider the fuel choice at this stage, since that has already been decided to be δ_4.

At Stage 2 we choose the best configuration for the CL subsystem maximizing profit associated with its production. For given values of t_x, t_y and M_{CL} ($\hat{s}_2$) and $conf_{CL}$ we can determine:

1. the production of heat and electricity for CL, and therefore the corresponding returns,

$$r_{CL}(t_x, t_y, M_{CL}, conf_{CL}) = p_E W_{E,CL}(\hat{s}_2, conf_{CL}) + p_Q Q_{p,CL}(\hat{s}_2, conf_{CL})$$

2. the steam rate requirements for turbines T1, T2 and T3, obtained through the transition function $T_2(t_x, t_y, M_{CL}, conf_{CL})$, and therefore the costs associated with steam production.

The objective function for Stage 2 is

$$f_2(t_x, t_y, M_{CL}) = max_{conf_{CL}} \Big(r_{CL}(\hat{s}_2, conf_{CL}) - c\,(m_{1B}, m_{2B}, m_{3B}, \delta_4(m_{1B}, m_{2B}, m_{3B})) \Big)$$

subject to the constraints

$$(m_{1B}, m_{2B}, m_{3B}) = T_3(\hat{s}_3, \delta_3(\hat{s}_3))$$

$$\hat{s}_3 = T_2(\hat{s}_2, conf_{CL})$$

At Stage 1, the best configuration for the TU subsystem is taken into consideration, with the purpose of maximizing profit obtained by the whole plant. The objective function for this stage depends on the state $\hat{s}_1 = (Q_p, t_{ret}, t_p)$ with which it is associated and coincides with Equation 8,

$$f_1(Q_p, t_{ret}, t_p) = max_{conf_{TU}} \Big(r_{CL}(\hat{s}_2, \delta_2(\hat{s}_2)) + r_{TU}(\hat{s}_1, conf_{TU}) - c\,(m_{1B}, m_{2B}, m_{3B}, \delta_4(m_{1B}, m_{2B}, m_{3B})) \Big)$$

subject to the constraints

$$(m_{1B}, m_{2B}, m_{3B}) = T_3(\hat{s}_3, \delta_3(\hat{s}_3))$$

$$\hat{s}_3 = T_2(\hat{s}_2, \delta_2(\hat{s}_2))$$

$$\hat{s}_2 = T_1(\hat{s}_1, conf_{TU})$$

3.1 Example: solving the dynamic problem

To clarify the solution algorithm we consider the example described in Figure 3. The last three stages of the optimization process are here represented with the purpose of finding the best path for state 0 at stage $N-2$. For simplicity of the example, for the stages beginning at 22:00 and 23:00, we assume just three possible states of the reservoir, even though the solution of the problem is more accurate if more states are considered.

The first step consists of associating with each arc the set of values of Q_p and t_p defined by the inverse of the transition function (Equations 2 to 5) for Q_p and t_p (t_{ret} is set to 60°C). The solution of the static problem for these values yields (Equation 8) the optimal hourly marginal profit for the arc considered (*MP*, Table 2). For the last stage there is only one arc for each state s that leads to the last state, consequently, the value of $f_N(s)$ associated to state s will be that of *MP* (first row, Table 3).

For each state s of the stage $N-1$, the three possible decisions (arcs) must be compared. With each one of them a value of MP_c can be associated, calculated as the sum of the *MP* for that arc and $f_N(s')$, where s' is the state at the beginning of stage N, resulting from the decision taken (MP_c, Table 2). The value of $f_{N-1}(s)$ is the maximum of the MP'_c for that state. $\delta_{N-1}(s)$ is the corresponding value of the optimal decision for Q_p and t_p. The same algorithm must be repeated for the state at stage $N-2$. The optimal path from this state to the end turns out to be: m $\rightarrow$ d $\rightarrow$ a.

The algorithm can be extended to a 24-hour period subject to additional constraints on electricity and heat hourly consumptions $W_E(n)$ and $Q_r(n)$. Typical results obtained for the optimal heat production $Q_p(n)$ and the state of the reservoir during the day are shown in Figures 4 and 5.

3.2 Example: solving the static problem

In this Section we explain with an example how the optimal value of *MP* can be associated with given values of Q_p, t_p and t_{ret}, taking into consideration some additional constraints that the user might impose. For instance, we clarify the reason why in the previous example the value of $MP = 27.16$ has been associated with arcs d, h and l, all characterized by the same values of Q_p, t_p and t_{ret}.

We suppose that the user requested a configuration of the plant where the waste-to-energy facility TU produces only electricity, boilers B1 and B2 use only fuel oil ($\sigma_1 = \sigma_2 = 100\%$) and boiler B3 warms 80% of its flow with coal and the remaining part with methane ($\sigma_3 = 20\%$, $\rho = 1$).

In this case, the solution of the optimization problem at Stage 4 is immediate since for each boiler there is only one possible value for the fuel choice, that chosen by the user, which is also the optimal solution. In other circumstances we would have to enumerate all possible values for the fuel choice and sort the best one. With reference to

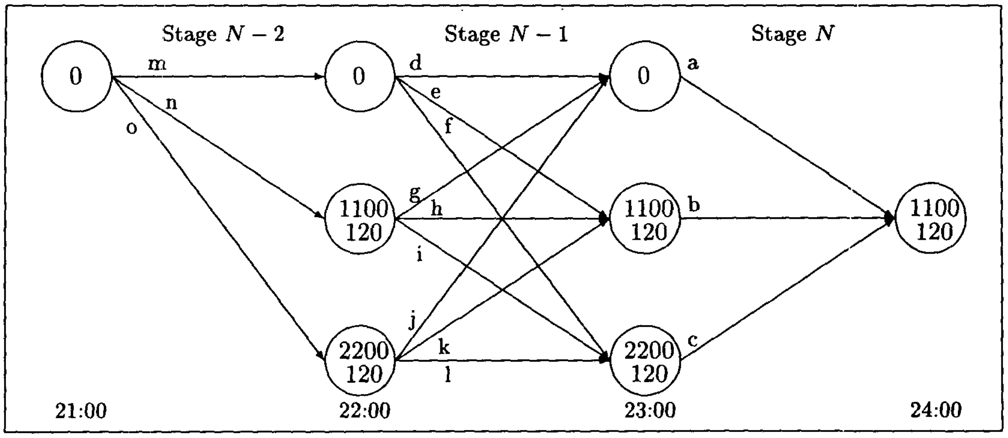

Fig. 3: Example.

	Q_p (MW)	t_p (°C)	MP (M£)	MP_c (M£)		Q_p (MW)	t_p (°C)	MP (M£)	MP_c (M£)		Q_p (MW)	t_p (°C)	MP (M£)	MP_c (M£)
a	233.05	120	28.43	28.43	b	156.05	114	24.94	24.94	c	79.05	110	19.03	19.03
d	201.38	116	27.16	55.59	e	278.38	120	29.97	54.91	f	355.38	120	32.09	51.12
g	124.38	114	22.97	51.40	h	201.38	114	27.16	52.10	i	278.38	120	29.97	49.00
j	47.38	120	14.58	43.01	k	124.38	114	22.97	47.91	l	201.38	116	27.16	46.19
m	251.33	114	29.08	84.67	n	328.33	120	31.40	83.50	o	405.33	120	33.24	81.15

Table 2: Arcs.

	$s = (M_{res}, t_{res})$		
Time	(2200,120)	(1100,120)	(0,-)
23:00 h	$f_N(s) = 19.03$ $\delta_N(s) = (79.05, 110)$ arc c	$f_N(s) = 24.94$ $\delta_N(s) = (156.05, 114)$ arc b	$f_N(s) = 28.43$ $\delta_N(s) = (233.05, 120)$ arc a
22:00 h	$f_{N-1}(s) = 47.91$ $\delta_{N-1}(s) = (124.38, 114)$ arc k	$f_{N-1}(s) = 52.1$ $\delta_{N-1}(s) = (201.38, 116)$ arc h	$f_{N-1}(s) = 55.59$ $\delta_{N-1}(s) = (201.38, 116)$ arc d
21:00 h			$f_{N-2}(s) = 84.67$ $\delta_{N-2}(s) = (251.33, 114)$ arc m

Table 3: States.

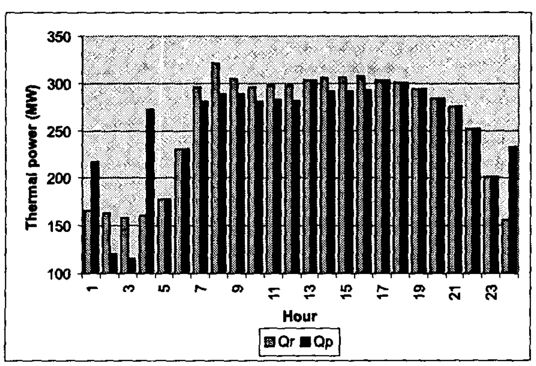

Fig. 4: Heat demand and production

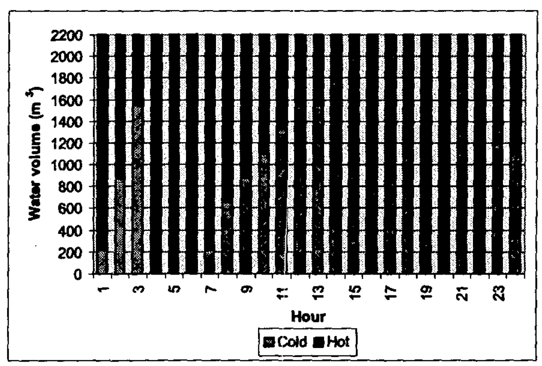

Fig. 5: State of the reservoir

(Figure 6), $f_4(\hat{s}_4)$ is

$$f_4(\hat{s}_4) = \begin{cases} 0.875 + 2.216 + 2.211 = 5.302 \\ \qquad \text{if } \hat{s}_4 = (20, 51, 47) \\ 0.875 + 1.563 + 2.911 = 5.349 \\ \qquad \text{if } \hat{s}_4 = (20, 36, 62) \end{cases}$$

$$\delta_4(\hat{s}_4) = (100\%, 100\%, 20\%, 1) \qquad \forall\, \hat{s}_4$$

In order to solve the optimization problem at Stage 4, it is necessary to split it into three separate problems (explosion).

At Stage 3, for several $\hat{s}_3$, the best value of τ is chosen. For instance, for $\hat{s}_3 = (20, 27, 71)$, the best τ is 52%, since it leads to the lower production cost, between 5.302 and 5.349. Therefore

$$f_3(20, 27, 71) = 5.302$$

$$\delta_3(20, 27, 71) = 52\%$$

The choice of $conf_{\mathrm{CL}}$ determines the heat and electricity production obtained by the CL subsystem and the amount of steam needed by each turbine (through the transition function). Table 4, compares two different options from state $\hat{s}_2 = (60, 116, 3200)$.

Therefore

$$f_2(60, 116, 3200) = max(11.261, 10.690) = 11.261$$

$$\delta_2(60, 116, 3200) = (94, 55\%)$$

There is only one possible choice for Stage 1, given by the user's constraint that TU shall produce only electricity. In this configuration, the electricity production rate is 54 MW, which contributes an additional 15.9 M£ to the profit,

$$f_1(201.38, 116, 60) = 11.261 + 15.9 = 27.161$$

4 Conclusions

We developed a model and a method, based on dynamic programming theory, capable of finding the optimal production schedule for the complex system of cogeneration facilities that serve the district heating network of the city of Brescia, Italy. A key component of the system is a heat reservoir, that allows a limited extent of decoupling of the heat request from its production.

The method assumes that simnple performance correlations and simplified models are available for each turbine group and boiler of the system, including the waste-to-energy facility and the heat-storage reservoir. Such correlations can be readily obtained by means of a structured statistical analysis of the available real performance data, and are not discussed in the paper since they are specific to each facility. We presented instead the general features and some technicalities of the optimization method that can be useful for its application to other facilities.

The method has been implemented into an efficient software that considers additional constraints and inputs given by the user and computes the optimal production schedule and load subdivision. For each hour of the day, the optimization method allows an efficient search of the choice of fuels and of the subdivision of the production load among the various units.

The software will be used for the Brescia system as a support for long-term choices related, for instance, to the best use of the waste-to-energy facility and also as a means to determine the optimal size of a new heat-storage reservoir.

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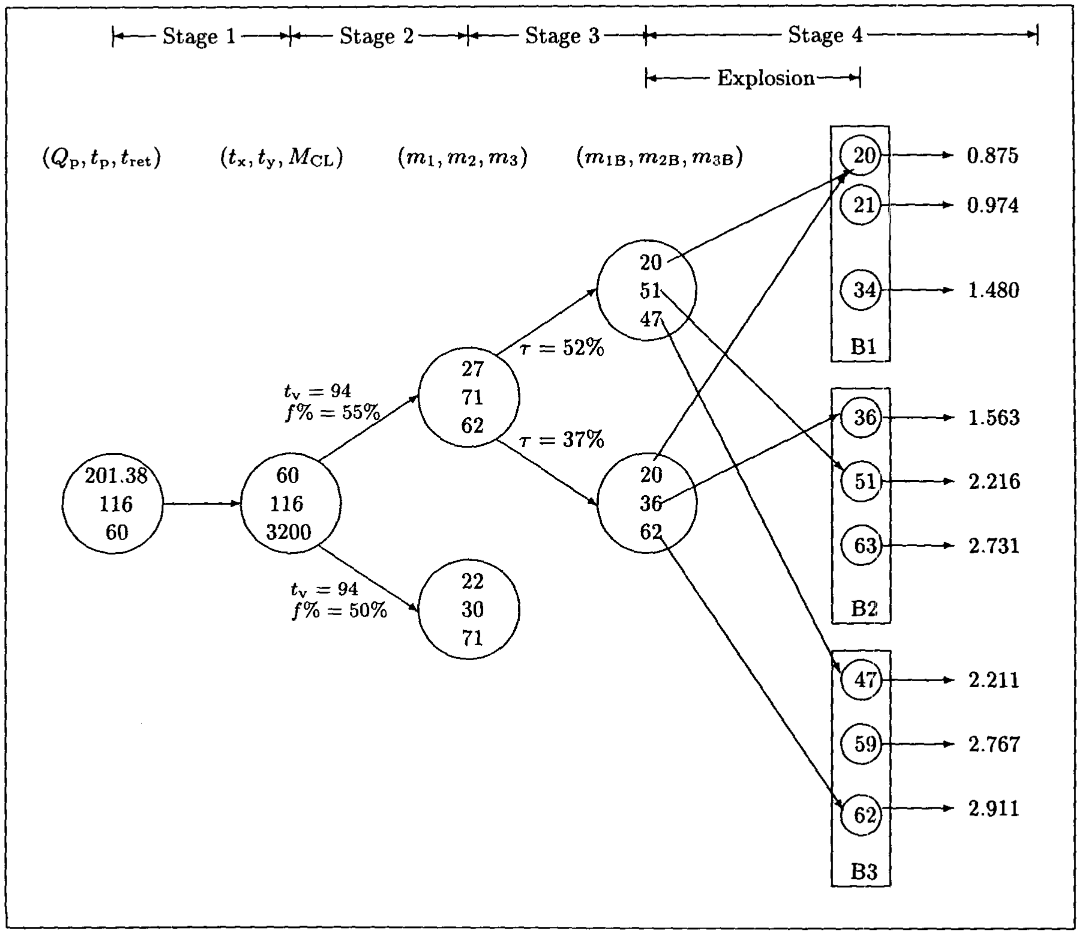

Fig. 6: States and transitions.

$conf_{CL}$	$W_{E,CL}(conf_{CL})$	$Q_{p,CL}(conf_{CL})$	$T_2(conf_{CL})$	r_{CL}	$c(\hat{s}_4, \delta_4(\hat{s}_4))$	$r_{CL} - c(\hat{s}_4, \delta_4(\hat{s}_4))$
$t_v = 94°C$ $f\% = 55\%$	89.511	201.38	(20,27,71)	16.563	5.302	11.261
$t_v = 94°C$ $f\% = 50\%$	86.213	201.38	(22,30,31)	16.103	5.413	10.690

Table 4: Stage 2