The Second Law from Locally Maximal Entropy Generation Quantum Dynamics

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Abstract. First, we briefly address the questions: "Is entropy defined for non-equilibrium states?" and "Is entropy an intrinsic property of matter?". Then, we focus on: "Is irreversibility an intrinsic feature of microscopic dynamics?" We answer this question by discussing the structure and main features of the nonlinear evolution equation proposed by this author to capture precisely how the essence of the Second Law of Thermodynamics could be embedded at the most fundamental level of description, i.e., the quantum level. The nonlinear equation generates a dynamical group providing a unique deterministic description of irreversible, conservative relaxation towards equilibrium from any non-equilibrium state, following a locally constrained, steepest entropy ascent, maximal entropy generation path. The Hatsopoulos-Keenan statement of the Second Law emerges as a general theorem about the Lyapunov stability of the equilibrium states of the proposed evolution equation. An appendix gives mathematical details and outlines other important theorems.

I consider the Hatsopoulos-Keenan statement of the Second Law ("HK2nd law" for brevity) [1] a major breakthrough in the history of the foundations of thermodynamics, a pioneering intuition about the role of stability at thermodynamic equilibrium, which opened up entirely new vistas, and new answers to at least three of the key questions we are asked to address in this symposium. Based on the HK2nd law, the school of thermodynamics founded by professor Keenan at MIT, developed very intriguing answers to these questions, between 1976 and 1991, somewhat in competition with the Brussels school [2].

IS ENTROPY DEFINED FOR NON-EQUILIBRIUM STATES?

My answer to this question is a definite and strong "yes". In fact, in Ref. [3] we have developed a rigorous axiomatic exposition of the principles of thermodynamics — that we have taught to engineering undergraduate and graduate students for over twenty years — that resolves the long standing problem of logical loops and non-rigorous definitions of the traditional heuristic expositions, and which includes a very precise, unambiguous and general definition of entropy valid also for nonequilibrium states, no matter how far from equilibrium.[1]

Many in this audience are still convinced, and too many textbooks and papers about Thermodynamics still state that entropy, for non-equilibrium states, is either not defined or it is not clear that it is defined. Yet, our definition has been available for over 15 years

[1] The essential steps of our definition are outlined in the paper by Zanchini and Beretta in this volume.

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[3] and it is based on simple axiomatic reasoning of the same kind used in most traditional presentations; *a posteriori*, it can actually be viewed as a straightforward generalization of the line of thought underlying the definition given by Fermi for equilibrium states [4]; in particular, it is entirely independent of any of the endless controversies and interpretational issues that surround the concept of entropy in statistical mechanics, information theory, etc. I am aware of no other rigorous, precise and general definitions that do not resort to any statistical mechanics modeling and reasoning.[2]

Nonequilibrium states play a key role in a variety of state-of-the-art mesoscopic and microscopic applications. Because of this role, thermodynamics today cannot anymore be considered "a dead subject" having to do with equilibrium only, like many used to think about thirty years ago.[3] Instead, it is well alive and actually starring at the forefront of many technological disciplines, as well as being still a challenge in probing our understanding of the foundations of science and engineering.

So, when developing the next generation of thermodynamics textbooks, or teaching and scientific articles, we ought to make an effort to convince ourselves, and then our readers, that entropy *is* well defined also for nonequilibrium states, and we do need this fact to understand a variety of phenomena and gain better control of irreversibility.

IS ENTROPY AN INTRINSIC PROPERTY OF MATTER?

By our engineering experience and intuition we would expect that the answer to this question is "yes", but this is not the currently prevailing physical (statistical mechanics) explanation. However, Hatsopoulos and Gyftopoulos [8] have shown that, without contradicting any of the known successful results of the prevailing physical theories, all that is needed to incorporate the Second Law at the fundamental level is to extend quantum mechanics, so that entropy emerges as an intrinsic property of matter [9]. One way to illustrate how this is done, is by considering a single two-level quantum system, a "qubit". If the qubit is non-interacting and non-correlated with anything else in the universe, the set of available quantum mechanical states maps one-to-one with the points on a unit sphere, the Bloch sphere. The key ansatz of the Hatsopoulos-Gyftopoulos theory ("HG ansatz, for brevity") is that the set of available quantum states is broader, and maps one-to-one with the unit ball, not just the unit sphere. In other words, *all* density operators, not just the idempotent ones, are assumed to represent distinct true individual quantum states of any uncorrelated (and separable [9]) microscopic system, where by microscopic system we mean each individual member of a *homogeneous ensemble* (homogeneous in

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[2] The attempt in Refs. [5, 6] does keep statistical mechanics modeling and reasoning away from the definition, but the restriction to equilibrium states is *declared* at the outset, and the requirement of *extensivity* actually restricts it even further to the domain of validity of what in [3, Ch.17] we call the *simple system model*, which is a good approximation only for macro- and mesoscopic systems.

[3] Perhaps even fifteen, judging from a review of our book [3] published in Nature in 1992 [7] that "hesitates" to give the book "a full-blooded recommendation" because our basic framework does not comprehend "a phase space in which points represent equilibrium states and curves represent quasistatic (that is, reversible) changes". By the way, "quasistatic" changes—being processes whereby the system evolves through sequences of stable equilibrium states—need *not* necessarily be reversible (this is easily concluded from the straightforward understanding of nonequilibrium states that we develop in our book).

the sense defined by von Neumann [9]). If this ansatz is valid, entropy is an intrinsic property of matter.

Notice the conceptual difference from quantum statistical mechanics, where a non-idempotent density operator indicates that the system is randomly picked out of a mixed *heterogeneous ensemble*. At the time of Ref. [8], the ansatz was felt as unacceptable and "adventurous" [10]. Today, in apparent contradiction with the HG ansatz, an emerging consensus in the quantum foundations community [11] accepts the existence of non-idempotent states for microscopic systems and single particles, but attributes it to the existence of classical correlations or quantum entanglement between the system and other systems in the universe, such as heat baths or anything else with which it has previously interacted. The current belief is that if all such correlated systems could be identified and included in the description, then the overall system would be in an idempotent (zero entropy) state.[4]

If however the correlated systems cannot be identified (because maybe they escaped far away) and the dynamics of the overall (albeit unknown) system is unitary, then no empirical method could possibly distinguish between the nonidempotency being intrinsic or due to correlations/entanglement. Therefore, the two possibilities—HG ansatz and "untraceable correlations ansatz"—turn out to be empirically indistinguishable; hence, they must be considered physically equivalent (in spite of the apparently profound conceptual difference). In other words, we must conclude that the contradiction between the two ansatzs is only apparent.

If instead, contrary to the prevailing view, the unitarity of the fundamental dynamics of uncorrelated and unentangled systems should turn out not to be a universal law, then there may exist ways to empirically test which of the two ansatzs is right.

This is one of the unresolved entropy challenges.

IS IRREVERSIBILITY AN INTRINSIC FEATURE OF MICROSCOPIC DYNAMICS?

Again, the current prevailing view about this question is: no, if by intrinsic dynamics you mean the fundamental law of causal evolution of any isolated, uncorrelated and unentangled system, then the dynamics is unitary and hence reversible. If such a system is a qubit, a two-level system, the states evolve linearly and unitarily in periodic motions

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[4] By this reasoning, the entire universe, which by definition includes everything—and hence nothing is left out to be correlated with—can only be in a pure state, and hence have zero entropy. But then, if cosmological estimates assign a nonzero entropy to the universe [12], we should infer that they either overlook the contribution of some parts, or they just refer to the sum of 'local' entropies, which in turn is exactly equal to the 'amount of correlation' that has accumulated so far between parts of the universe, due to their past interactions. Indeed, a measure of the 'amount of correlation' between two subsystems A and B of a composite AB in state ρ is given by the nonnegative-definite *correlation functional* (we defined it in [13, Eq. 47]), $\sigma_{AB}(\rho) = \mathrm{Tr}(\rho\ln\rho) - \mathrm{Tr}_A(\rho_A\ln\rho_A) - \mathrm{Tr}_B(\rho_B\ln\rho_B)$ which takes also the equivalent forms: $\sigma_{AB}(\rho) = [s(\rho_A) + s(\rho_B) - s(\rho)]/k_\mathrm{B} = \mathrm{Tr}[\rho(P_{\rho>0}\ln\rho - P_{\rho_A>0}\ln\rho_A\otimes I_B - I_A\otimes P_{\rho_B>0}\ln\rho_B)] = \langle S(\rho_A)\otimes I_B + I_A\otimes S(\rho_B) - S(\rho)\rangle/k_\mathrm{B}$, where $s(\rho) = -k_\mathrm{B}\mathrm{Tr}(\rho\ln\rho)$, $S(\rho) = -k_\mathrm{B}P_{\rho>0}\ln\rho$, and $P_{\rho>0}$ is the projection operator onto the range of ρ. If the composite system AB has zero entropy, $s(\rho) = 0$, it is clear that $\sigma_{AB}(\rho)$ equals to the sum of the local entropies (divided by k_B).

on the unit sphere according to the Schrödinger equation.

The problem I undertook in my doctoral thesis [14] was to "design" an extension of the Schrödinger equation to the interior of the unit ball, compatible with the HK2nd law, via the HG ansatz. Indeed, my steepest-entropy-ascent (SEA) or locally-maximal-entropy-generation (LMEG) dynamical law entails the HK2nd law as a *theorem about the dynamical (Lyapunov) stability of the equilibrium states*[5] (math details in the Appendix).

The equation I proposed implements the competition of two orthogonal tendencies: one is the usual Hamiltonian tendency to produce a unitary evolution of the density operator; the other is a "dissipative" tendency to spontaneously redistribute *probabilities* by which the system rearranges how its energy is distributed among the available eigenmodes. This spontaneous energy-load sharing redistribution follows the path of SEA or LMEG compatible with conservation of energy (and number of constituents) as well as separability and no-signaling conditions for composite systems (see Appendix).

When my equation of motion is written for a qubit, the dissipative (but conservative) redistribution tendency vanishes on the Bloch surface of the unit ball, where the entropy is zero, and we recover Schrödinger's unitary equation of motion of pure states. However, inside the unit ball, where density operators are non-idempotent and the entropy is not zero my dynamical law is largely nonlinear and—in a context where mixed density operators represent intrinsic entropy—it incorporates intrinsic irreversibility at the fundamental microscopic level [15]. This equation achieves what the Brussels school [2] searched for years with no success.

For a d-level system, a "qudit", if focus [16] for simplicity on the density operators that commute with the Hamiltonian, only the nonlinear redistribution term is active, and the eigenvalues of ρ represent the relative mean occupation of the system's energy levels, i.e., they measure the degree of involvement of each energy level in carring the overall energy load. Then, the *entropy*, $-k_{\rm B}{\rm Tr}(\rho\ln\rho)$, takes on a very clear and concrete *physical* meaning: *it measures the degree of sharing by which the active energy eigenmodes contribute to carrying the system's overall energy.* The entropy is zero when only one mode carries all the energy: no sharing. The maximal sharing obtains instead when the load is canonically distributed among all the system's eigenmodes. Physically, the equation is designed to describe a *local*[6] and spontaneous[7] tendency to rearrange noncanonical energy-load distributions until they become either *canonical* (maximal sharing among *all* the energy eigenmodes; a feature of stable equilibrium states only) or *partially canonical* (maximal sharing among *a subset* of active energy eigenmodes,

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[5] There is one and only one stable equilibrium state for each possible set of values of: (i) the (mean) energy ${\rm Tr}[H(V)\rho]$; (ii) the parameters V of the Hamiltonian operator $H(V)$; and (iii) the (mean) amounts of elementary constituents, ${\rm Tr}(N_i\rho)$, where N_i, for $i=1,2,\ldots,r$, is the number-of-particles operator for the i-th kind of particles in the system.

[6] I say *local*, because the terms in the equation responsible for driving the energy redistribution contain only local projections [see Eqs. (6)-(7)] of the overall system operators H, N_i, ρ, and $S=-k_{\rm B}P_{\rho>0}\ln\rho$.

[7] I say *spontaneous* because the dissipative terms in the equation of motion represent, in its original context, an intrinsic tendency, internal to every degree of freedom and not due to interactions with heat baths and the like, therefore active even if the system is isolated, except when ρ belongs to the very restricted subset of *non-dissipative states*, which includes all idempotent states [see Eq. (4)].

while the remaining ones do not participate at all; a feature of unstable and metastable equilibrium).[8]

For every density operator, the equation generates a unique trajectory well-defined not only forward in time, but also backwards in time [16]. In addition to the Second Law, it entails other important theorems and geometrical features, including Onsager reciprocal relations and fluctuation-dissipation relations [17], and time-energy and time-entropy uncertainty relations [18].

Perhaps unexpected, another interesting result obtains from the structure of the equation of motion for a composite of two non-interacting subsystems: the local evolution depends on the correlations that have built up in past interactions. This is in contrast with the usual assumption of strictly Markovian local dynamics, whereby if Alice does not interact with Bob then $\dot{\rho}_A$ must be a function of ρ_A only, as is the case under unitary dynamics. Instead, under our nonunitary evolution, existing correlations between Alice and Bob may influence their future, without this fact producing any paradox or violation of the no-signaling condition (details in the Appendix)[9].

CONCLUDING REMARK

In past and recent years, many authors in a variety of contexts [20], have observed that irreversible natural phenomena at all levels of description seem to obey a principle of general and unifying validity that has been named "maximum entropy production principle". This principle is in tune with our steepest-entropy-ascent dynamical ansatz, in which, at least at the quantum level of description, it finds a fundamental justification and a sound mathematical formulation. We originally conceived it to capture precisely how the essence of the Second Law could be embedded at the most fundamental level of description. But in view of the resulting intriguing general mathematical and geometrical features it has in the quantum context, it should also find immediate application outside of quantum thermodynamics, in a variety of fields where a well-behaved evolution equation may serve as a useful phenomenological kinetic and dynamical modeling tool.

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[8] The null eigenspace of ρ remains invariant. Hence, the initially zero eigenvalues of ρ remain zero at all times. By this very important feature, energy eigenmodes that are not already active cannot be activated by the redistribution term which, in particular, cannot evolve pure states into mixed states. In classical terms, this entropy generation via redistribution occurs only by smoothing out the density over the filled region of phase space, not by diffusing it out into new volume; and this is so, i.e., phase-space volume is conserved, also when Hamiltonian evolution moves, stretches and convolutes the filled phase-space region.

[9] Note that our Eq. (5) is not of the form considered by Gheorghiu-Svirschevski in her Eq. (3) in this volume, and is not affected by the problem outlined in Footnote 3 therein. Indeed, as shown in the Appendix, it does entail strong separability conditions.

Artemis, Vasso, Maro, and Rena Gyftopoulos for their so warmest embrace to Giulia and me during our stay in Boston for the 2007 fall term. Finally, I thank the Department of Mechanical Engineering at MIT for inviting me to give a special project course on Quantum Thermodynamics and all the members of the Hatsopoulos Microfluids Lab for their kind hospitality.

APPENDIX. LOCALLY-MAXIMAL-ENTROPY-GENERATION QUANTUM DYNAMICS

Among the several equivalent forms that SEA or LMEG quantum dynamics may take for a single particle or a localized open field of particles [14, 13], I present here the most compact [19] for an *adiabatic* system[10]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}[H,\rho] + \frac{1}{2k_\mathrm{B}\tau}\{\Delta M,\rho\} \tag{1}$$

where[11] $\tau = \tau(\rho)$ is a yet unspecified positive-definite functional describing the characteristic time of the internal redistribution mechanism, and the *dissipation operator* $\Delta M = \Delta M(\rho, H, \mathbf{N})$ is defined by

$$\Delta M = \Delta S + [\boldsymbol{\mu}\cdot\Delta\mathbf{N} - \Delta H]/\theta \tag{2}$$

where $\theta = \theta(\rho)$ and $\boldsymbol{\mu} = \boldsymbol{\mu}(\rho) = \{\mu_1(\rho),\ldots,\mu_r(\rho)\}$ are the set of real nonlinear functionals defined for each ρ by the solution of the system of $1+r$ linear equations (with nonlinear coefficients),[12] $\langle\Delta S\Delta H\rangle\,\theta + \sum_{i=1}^{r}\langle\Delta N_i\Delta H\rangle\,\mu_i = \langle\Delta H\Delta H\rangle$ and $\langle\Delta S\Delta N_j\rangle\,\theta + \sum_{i=1}^{r}\langle\Delta N_i\Delta N_j\rangle\,\mu_i = \langle\Delta H\Delta N_j\rangle\ \ \forall j$, which warrants that $\langle\Delta H\Delta M\rangle = 0$ and $\langle\Delta N_j\Delta M\rangle = 0\ \forall j$, and hence that the dissipative term "conserves" the mean energy $\langle H\rangle = \mathrm{Tr}(\rho H)$ and the mean number of particles $\langle\mathbf{N}\rangle = \mathrm{Tr}(\rho\mathbf{N})$.[13] The entropy, $\langle S\rangle = \mathrm{Tr}(\rho S) = -k_\mathrm{B}\mathrm{Tr}(\rho\ln\rho)$, changes at the rate

$$\frac{\mathrm{d}\langle S\rangle}{\mathrm{d}t} = \frac{1}{k_\mathrm{B}\tau}\langle\Delta M\Delta M\rangle \tag{3}$$

which is clearly non-negative definite,[14] and is zero only for the *non-dissipative states*

$$\rho_\mathrm{nd} = \frac{B\exp[(\boldsymbol{\mu}_\mathrm{nd}\cdot\mathbf{N} - H)/k_\mathrm{B}\theta_\mathrm{nd}]B}{\mathrm{Tr}\left(B\exp[(\boldsymbol{\mu}_\mathrm{nd}\cdot\mathbf{N} - H)/k_\mathrm{B}\theta_\mathrm{nd}]\right)} \tag{4}$$

where B is any projection operator ($B^2 = B$), either constant in time or evolving unitarily according to $\mathrm{d}B/\mathrm{d}t = -i[H,B]/\hbar$. If B is a one-dimensional projector, this coincides with the standard Schrödinger dynamics of pure states, which emerge therefore as boundary limit cycles of our dynamics. The only stable equilibrium states obtain when $B = I$. Then, Eq. (4) yields a *grand-canonical* density operator,

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[10] By *adiabatic* we mean that the Hamiltonian operator H has some time-dependent parameters $\boldsymbol{\beta}$ which describe the effects of external forces. If the parameters are time-independent, the system is *isolated*.

[11] Notation: $[\cdot,\cdot]$ and $\{\cdot,\cdot\}$ are the usual commutator and anticommutator, H is the Hamiltonian operator of the system (not necessarily independent of time t), $\hbar$ is the reduced Planck constant, k_B the Boltzmann constant, $S = -k_\mathrm{B}P_{\rho>0}\ln\rho$ our "entropy operator"[4] (well defined for any ρ), and for any operator $F = S, H, N_i, \ldots$, $\langle F\rangle = \mathrm{Tr}(\rho F)$, $\Delta F = F - I\langle F\rangle$, $\langle\Delta F\Delta G\rangle = \mathrm{Tr}[\rho\{(\Delta F),(\Delta G)\}]/2 = \mathrm{Tr}(\rho\{F,G\})/2 - \langle F\rangle\langle G\rangle$.

[12] By Cramer's rule, the solution of this linear system of equations can be expressed explicitly as a ratio of determinants. This yields the explicit forms (and the geometrical interpretations in terms of Gram determinants) given in our original references and, for example, Eq. (14) by Bedeaux in this volume.

[13] For a canonical system, $\langle\Delta\mathbf{N}\rangle = 0$, θ is given for all states by $\theta = \langle\Delta H\Delta H\rangle/\langle\Delta S\Delta H\rangle$; moreover, $\theta^2 \geq \langle\Delta H\Delta H\rangle/\langle\Delta S\Delta S\rangle$ with equality only if the state is non-dissipative, Eq. (4).

[14] It is also *maximal* with respect to all possible evolutions of ρ which conserve $\langle H\rangle$ and $\langle\mathbf{N}\rangle$ [21].

which reduces to a *canonical* one if in addition $\langle \Delta \mathbf{N} \rangle = 0$, or a *micro-canonical* if we also have a fully degenerate Hamiltonian, $\langle \Delta H \rangle = 0$.

As $t \to +\infty$, the state operator $\rho(t)$, i.e., the solution of Eq. (4) for any initial $\rho(0)$, approaches a non-dissipative operator of form (4), θ and $\boldsymbol{\mu}$ approach smoothly the corresponding *thermodynamic equilibrium* (or partial equilibrium) *temperature* $\theta_{\rm nd}$ and *chemical potentials* $\boldsymbol{\mu}_{\rm nd}$ respectively. In this sense, therefore, θ and $\boldsymbol{\mu}$ can be interpreted as *nonequilibrium extensions* of the temperature and the chemical potentials.

When it comes to composite systems, the nonlinearity of the dynamical law imposes that the structure of the interactions and the "internal constraints" between subsystems must be described not only through the Hamiltonian operator, but also through the structure of the dynamical equation itself. Suppose Alice and Bob are the two elementary subsystems, A and B, of an adiabatic system. Each subsystem is either a single particle or a localized open field of particles. Alice and Bob may be either interacting ($H = H_A \otimes I_B + I_A \otimes H_B + V_{AB}$) or noninteracting ($V_{AB} = 0$), and either correlated/entangled[4] [$S(\rho) \neq S(\rho_A) \otimes I_B + I_A \otimes S(\rho_B)$] or uncorrelated [$\sigma_{AB}(\rho) = 0$]. Then LMEG dynamics takes the form [13, 19]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}[H,\rho] + \frac{1}{2k_{\rm B}\tau_A}\{(\Delta M)^A, \rho_A\} \otimes \rho_B + \frac{1}{2k_{\rm B}\tau_B}\rho_A \otimes \{(\Delta M)^B, \rho_B\} \tag{5}$$

where τ_A, τ_B are positive-definite characteristic times of local internal redistribution,[15] and $(\Delta M)^A$, $(\Delta M)^B$ their respective *local dissipation operators* defined by

$$(\Delta M)^A = (\Delta S)^A + \left[\boldsymbol{\mu}_A \cdot (\Delta \mathbf{N})^A) - (\Delta H)^A\right] / \theta_A \quad \text{with } (\Delta F)^A = \mathrm{Tr}_B[(I_A \otimes \rho_B)F] \; \forall F \tag{6}$$

$$(\Delta M)^B = (\Delta S)^B + \left[\boldsymbol{\mu}_B \cdot (\Delta \mathbf{N})^B) - (\Delta H)^B\right] / \theta_B \quad \text{with } (\Delta F)^B = \mathrm{Tr}_A[(\rho_A \otimes I_B)F] \; \forall F \tag{7}$$

Here θ_J and $\boldsymbol{\mu}_J$ (for $J = A,B$) are defined for each ρ by the solution of the system of $1+r$ linear equations, $\langle (\Delta S)^J (\Delta H)^J \rangle \, \theta_J + \sum_{i=1}^r \langle (\Delta N_i)^J (\Delta H)^J \rangle \, \mu_{iJ} = \langle (\Delta H)^J (\Delta H)^J \rangle$ and $\langle (\Delta S)^J (\Delta N_j)^J \rangle \, \theta_J + \sum_{i=1}^r \langle (\Delta N_i)^J (\Delta N_j)^J \rangle \, \mu_{iJ} = \langle (\Delta H)^J (\Delta N_j)^J \rangle \quad \forall j$, which warrants that $\langle (\Delta H)^J (\Delta M)^J \rangle = 0$ and $\langle (\Delta N_j)^J (\Delta M)^J \rangle = 0 \; \forall j$, and hence that each local dissipative term separately "conserves" the overall system's mean energy $\langle H \rangle = \mathrm{Tr}(\rho H)$ and mean number of particles $\langle \mathbf{N} \rangle = \mathrm{Tr}(\rho \mathbf{N})$. The overall system's rate of entropy change is

$$\frac{\mathrm{d}\langle S \rangle}{\mathrm{d}t} = \frac{1}{k_{\rm B}\tau_A}\langle (\Delta M)^A (\Delta M)^A \rangle + \frac{1}{k_{\rm B}\tau_B}\langle (\Delta M)^B (\Delta M)^B \rangle \tag{8}$$

where each subsystem's contribution is nonnegative definite.

If Alice and Bob come to interact, the Hamiltonian evolution during the interaction builds up correlations that survive even after they separate and loose touch completely. When that happens, the so called *no-signaling condition* requires that the evolution of Alice should become independent of whatever happens to Bob, and viceversa. This is reflected in the local structure of Eq. (5) and in particular of operators $(\Delta S)^A$, $(\Delta S)^B$, $(\Delta H)^A$, $(\Delta H)^B$, etc. Despite the nonlinearity, this structure of the non-Hamiltonian terms in the equation prevents "no-signaling" violations, in that it satisfies *strong (and weak) separability* conditions.

Indeed, denoting by $\dot{\rho}_{AB}(\rho, H, \mathbf{N})$ the rhs of Eq. (5), it is easy to show that it satisfies a strong separability condition, i.e., for any ρ and any H_A, H_B, $\mathbf{N}_A$, $\mathbf{N}_B$,

$$\mathrm{Tr}_B[\dot{\rho}_{AB}(\rho, H_A \otimes I_B + I_A \otimes H_B, N_{iA} \otimes I_B + I_A \otimes N_{iB} \; \forall i)] = f_A[(\Delta S)^A, H_A, \mathbf{N}_A] \tag{9}$$

$$\mathrm{Tr}_A[\dot{\rho}_{AB}(\rho, H_A \otimes I_B + I_A \otimes H_B, N_{iA} \otimes I_B + I_A \otimes N_{iB} \; \forall i)] = f_B[(\Delta S)^B, H_B, \mathbf{N}_B] \tag{10}$$

which, of course, imply also the weak separability condition, i.e., $f_A = \dot{\rho}_A(\rho_A, H_A, \mathbf{N}_A)$ and $f_B = \dot{\rho}_B(\rho_B, H_B, \mathbf{N}_B)$ whenever $\rho = \rho_A \otimes \rho_B$. Conditions (9) and (10) prevent locality problems by guaranteeing that if Alice and Bob are noninteracting, no changes in the Hamiltonian of Bob (e.g.,

[15] To guarantee strong separability, the positive-definite characteristic times must either be constants or local functionals of the form ($J = A,B$) $\tau_J = \tau_J[(\Delta S)^J, (\Delta H)^J, (\Delta \mathbf{N})^J]$.

switching on a measurement apparatus on Bob) can affect the local evolution of Alice, and viceversa. By Eq. (9), the rate of change of Alice's reduced density operator ρ_A does not depend on H_B and, therefore, none of Alice's local observables (the functionals of ρ_A) can be affected by whatever happens to B, no faster-than-light communication can occur between B and A, even if Alice and Bob are correlated or entangled due to previous interactions. But existing correlations do influence their local evolutions, which therefore are not Markovian in that they do not depend only on their respective local (reduced) states ρ_A and ρ_B.

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