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# Novel approach for fair allocation of primary energy consumption among cogenerated energy-intensive products based on the actual local area production scenario

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ABSTRACT

Multi-generation facilities are almost always part of a local production scenario, i.e., a local area (district, city, regional, national, interstate) energy system providing end users with electricity, residential heating or air-conditioning, industrial process steam, desalinated water, and/or other energy-intensive products. Because of the growth of energy consumption and environmental concerns, local, national, and international regulations and standards tend to incorporate and enforce methods for energy and environmental rating of the end uses of primary energy. Important to such methods, is the definition of fair criteria to allocate fuel consumption among cogenerated products. Allocation based on prescribed primary energy factors for each product corresponding to the average efficiencies of separate production facilities may result in unfair figures and inconsistencies which become increasingly important as cogeneration gains higher fractions of the local energy market. To overcome this problem, we propose a slightly more elaborate, but self-consistent method whereby the allocation is adaptive and self-tuned to the local energy scenario. For heat and power cogeneration, we propose to allocate fuel consumption on the basis of the average primary energy factors for electricity and heat in the given local area including the cogeneration facility of interest. We call it the Self-Tuned Average-Local-Productions Reference (STALPR) method.



## 1. Introduction

The question of how fuel consumption should be allocated between the different energy products of a cogeneration or multi-generation facility has deep roots in the history of Thermodynamics. For example, in the 1930s it prompted Keenan [1–3] and others (see [4] for a full set of references and an historical perspective) to develop the general concept of thermodynamic availability today better known as exergy.

In recent years, the same question has reached the legislating agenda of national and international energy agencies in charge of regulating local energy planning and policymaking using free energy market mechanisms. For example, the 2007 European standard EN 15316-4-5 [5] provides a method for comparing and rating the efficiencies of different residential heating systems, including small and large cogeneration-based district heating systems [6,7]. In a growing number of countries, the energy (and environmental impact) rating of a residential building is mandatory in any transaction, thus affecting its economic value and permit process.

Companies and government agencies have adopted various methods to allocate the primary energy consumption of an energy facility between its different products. Often the same allocation scheme is adopted also to allocate carbon dioxide and other emissions among the different products. Fuel consumption has been allocated in proportion to energy, exergy, economic value of the products as well on the basis of a prescribed reference scenario of separate productions [8–19]. A comprehensive review of the rationale of such allocation schemes and their relevant implications can be found in [9]. So far, none of these methods has obtained "universal" acceptance. The practical need for a method that allocates fuel savings obtained via cogeneration in a fair way, calls for further rational analysis [9,16,20].

Recent regulations are shifting from allocation based on the incremental fuel consumption with respect to either the production of electricity only or of heat only, to allocation based on sharing the fuel savings on the basis of prescribed primary energy factors for electricity and heat usually corresponding to the average

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efficiencies of separate production facilities. The latter allocation method is fairer than the former in that it attempts to assign a fair share of the cogeneration benefits to both cogenerated products, as opposed to just one of the two. However, it does so with respect to a prescribed reference set of separate production efficiencies, and as a result the method results in unfair, distorted figures arising from an inconsistency which becomes increasingly important as cogeneration gains higher fractions of the energy market in a given local area.

In this paper, we present a self-consistent method in which the allocation parameters are self-tuned by taking into account the energy scenario of the given local area of interest. We consider this approach a contribution towards a "fairer" representation of the shares of fuel consumption with respect to the procedure followed in EN 15316-4-5 [5] based on the standard separate productions scenario. As noted in [21], in view of the broad economic impact of such regulations, if the regulatory principles overlook the "fairness" issue, they may result in market distortions which may for example improperly discourage investments in district heating systems fed by combined heat and power systems.

Finally, we emphasize that, although this paper focuses on heat and power cogeneration, the principle of self-tuning via average-local area parameters we introduce in order to obtain a "fair" allocation scheme can be readily generalized to deal with any different mix of products of a generic polygeneration facility, such as, for example, the case of combined production of power and desalinated water [22–25] and any other multi-product systems typically considered in Life Cycle Analysis [26,27]. Most importantly, as we will argue in a forthcoming paper, this methodology may assume particular relevance if applied to the allocation criteria for carbon dioxide, considering that the EU Commission is revising the regulatory principles guiding the development of the EU emission allowance trading scheme [21,28,29].

Wherever possible, we adopt a notation as close as possible to that used in EN 15316-4-5 [5].

The paper is organized as follows. In Section 2, we define the "allocation problem", review the available methods to define primary energy factors of cogenerated products, and discuss their drawbacks. In Section 3, we propose a method to overcome such drawbacks, first for the simplest case of a local area with only one cogeneration power plant, then for a general case with multiple cogeneration facilities. In Section 4, we analyze the results obtained using the new method for a particular heat and power example and compare them with the results obtained using the traditional methods. In Section 5, we further generalize the formulation to define the proposed fair allocation of primary energy consumption in multiple facilities of combined production of multiple goods. In Section 6, we draw our conclusions. In the Appendix, we provide a detailed analysis of the interrelations between the key parameters of the proposed method in the simplest case of only one cogeneration facility, showing how the differences between our allocation scenario and the traditional one grow as cogeneration takes higher shares of the local energy production.

## 2. Allocation problem definition

Current regulations originated when cogeneration (or combined heat and power, CHP) was in its early stages of penetration in the energy market, and cogenerated heat and electricity were still minor fractions of the overall heat and electricity production in a given system. To fix ideas, in this section we consider a single-fuel CHP facility that on a yearly basis consumes $E_{F,chp}$ of fuel energy (based on lower heating value) and delivers $E_{el,chp}$ of electrical energy and $E_{Q,chp}$ of thermal energy through a district heating network. We assume that the CHP facility is part of a certain area – that we call the *local area of interest* – in which the heat and power needs are supplied by *n* separate production electricity plants and *m* separate production heat plants. A sketch of the local area of interest and the power plants included therein is represented in Fig. 1.

For the cogeneration plant, we denote by $f_{el,chp}$ the primary energy factor for the cogenerated electricity, $f_{Q,chp}$ the primary energy factor for the cogenerated thermal energy, and by $f_{F,chp}$ the primary energy factor for the fuel used by the facility. Similarly, for the *i*-th separate production facilities we denote by $f_{el,sep,i}$ and $f_{Q,sep,i}$, respectively, the primary energy factors for the separately-produced electricity and thermal energy, and by $f_{F,el,sep,i}$ and $f_{F,Q,sep,i}$ the primary energy factor for the fuel used by the respective facility.

We recall that the "primary energy factor" of a given good is defined as the amount of primary energy that is currently consumed to produce a unit amount of that good, taking into consideration all processes in its life cycle. For example, the primary energy factor of electricity in the US in 1967 was estimated to be 3.8 [30], Directive 2006/32/EC proposed a value of 2.5 based on average

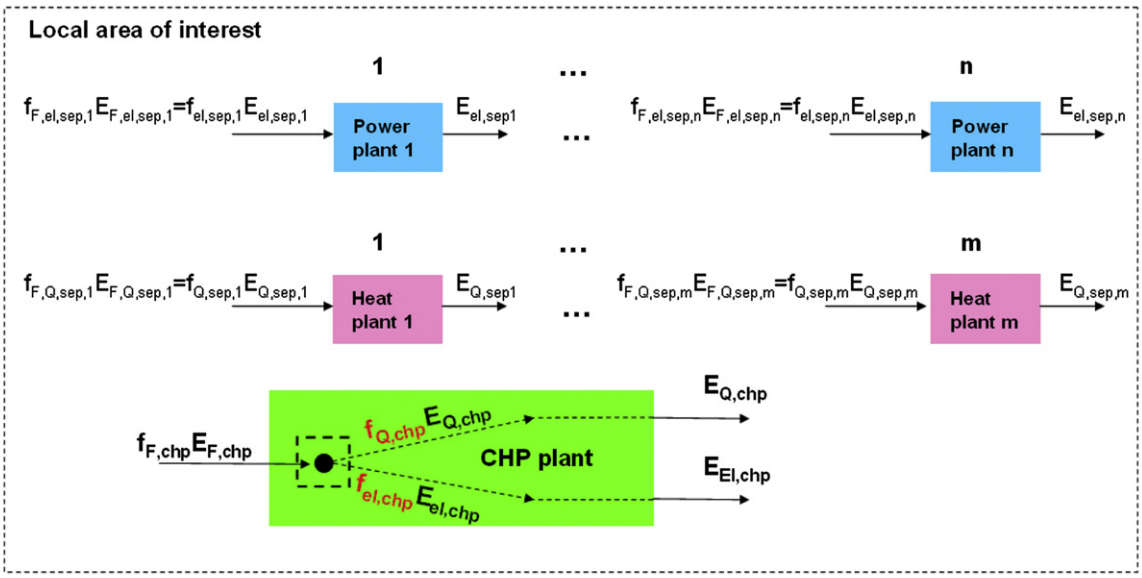


**Fig. 1.** Schematic representation of a local area of interest with a single cogeneration facility.

European production efficiency in 2003, but also suggested that a very different figure may apply for nations with high percentage of hydroelectricity or nuclear power; EN 15316-4-5 [5] suggests 2.8. Again, the primary energy factor suggested for natural gas delivered at power plant inlet in Europe is 1.1 [5], to take into account the pumping power expenditure for long-distance pipelining, which accounts for a primary energy consumption of about 10% that of the transported gas.

For instance, in the case of a separate production power plant that converts the fuel energy $E_{F,el,sep}$ (based on lower heating value) into electrical energy $E_{el,sep}$, the primary energy factor for electricity is defined by

$$f_{el,sep} = \frac{f_{F,el,sep} E_{F,el,sep}}{E_{el,sep}}$$

where $f_{F,el,sep}$ is the primary energy factor for the fuel used by the power plant. As indicated in Fig. 1, we may also write this relation as $f_{el,sep} E_{el,sep} = f_{F,el,sep} E_{F,el,sep}$. Of course, the primary energy factor $f_{el,sep}$ is related to the energy conversion efficiency $\eta_{el,sep}$ of the power plant, defined as the ratio $\eta_{el,sep} = E_{el,sep}/E_{F,el,sep}$, therefore,

$$f_{el,sep} = \frac{f_{F,el,sep}}{\eta_{el,sep}}$$

Similarly, in the case of a separate production heat facility that converts the fuel chemical energy $E_{F,el,sep}$ (based on the lower heating value) into thermal energy $E_{Q,sep}$, the primary energy factor for heat is defined by

$$f_{Q,sep} = \frac{f_{F,Q,sep} E_{F,Q,sep}}{E_{Q,sep}}$$

where $f_{F,Q,sep}$ is the primary energy factor for the fuel used by the heat facility. As indicated in Fig. 1, we may also write this relation as $f_{Q,sep} E_{Q,sep} = f_{F,Q,sep} E_{F,Q,sep}$. Of course, for a heat facility consisting of a furnace, the primary energy factor $f_{Q,sep}$ is related to the energy conversion efficiency $\eta_{Q,sep}$ of the furnace, defined as the ratio $\eta_{Q,sep} = E_{Q,sep}/E_{F,Q,sep}$, therefore,

$$f_{Q,sep} = \frac{f_{F,Q,sep}}{\eta_{Q,sep}}$$

Again, for a heat facility consisting of a heat pump operated on separately produced electricity, the primary energy factor $f_{Q,sep}$ is related to the coefficient of performance $COP_{Q,sep}$ of the heat pump, defined as the ratio $COP_{Q,sep} = E_{Q,sep}/E_{el,sep}$, and the primary energy factor $f_{el,sep}$ of the separately produced electricity, therefore

$$f_{Q,sep} = \frac{f_{F,el,sep}}{COP_{Q,sep}}$$

The intent of our analysis is to select a reasonable (fair) rule to determine how the primary energy consumption $f_{F,chp} E_{F,chp}$ of the cogeneration facility should be allocated between the two cogenerated products, i.e., how to split it into the two terms $f_{el,chp} E_{el,chp}$ and $f_{Q,chp} E_{Q,chp}$. Thus, the terms $f_{el,chp}$ and $f_{Q,chp}$, evidenced in red in Fig. 1, represent the two unknowns of this "fair allocation" problem.

However, they are not independent of one another due to the obvious constraint that the allocation rule itself must conserve primary energy, therefore, it must satisfy the primary energy balance over the dotted square shown in the Fig. 1,

$$f_{F,chp} E_{F,chp} = f_{el,chp} E_{el,chp} + f_{Q,chp} E_{Q,chp} \tag{1}$$

or, equivalently, the condition $\alpha_{el,chp} + \alpha_{Q,chp} = 1$ where the primary energy allocation fractions are defined as follows,

$$\alpha_{el,chp} = \frac{f_{el,chp} E_{el,chp}}{f_{F,chp} E_{F,chp}} \quad \text{and} \quad \alpha_{Q,chp} = \frac{f_{Q,chp} E_{Q,chp}}{f_{F,chp} E_{F,chp}} \tag{2}$$

As a result, the various allocation methods may be characterized by imposing some reasonable relation between $f_{Q,chp}$ and $f_{el,chp}$, which may be expressed in the generic form

$$f\left(f_{el,chp}, f_{Q,chp}, \text{"other parameters of the local area"}\right) = 0 \tag{3}$$

It is worth noting that all the existing methods neglect the influence of the "other parameters of the local area". In Sections 2.1–2.3 we briefly review the main existing allocation methods based on the primary energy factors that characterize a reference scenario of separate production, as they represent the rationale from which the 2007 European standard EN 15316-4-5 [5] is conceived. In Section 2.4 we comment on their inadequacies which motivate the development of the new method we propose in the rest of the paper. In Section 2.5 we briefly discuss the alternative exergy-based allocation method based on the primary energy factors that would characterize a hypothetical reference scenario of thermodynamically reversible production. Table 1 summarizes the results of this comparison.

### 2.1. Incremental electricity-centered reference (IECR)

According to this (obsolete) method, Eq. (3) is set as

$$f_{el,chp}^{IECR} = f_{el,sep} \tag{4}$$

so that by combining Eqs. (1) and (4) we obtain

**Table 1**

Heat and electricity primary energy factors based on the different allocation methods. Results in the fifth and sixth columns refer to the case study defined by the following parameters (details in Sections 2 and 3): $E_{F,chp} = 1000$ GWh, $f_F = 1.1$, $E_{el,chp} = 347$ GWh, $E_{Q,chp} = 350$ GWh, $f_{el,sep} = 2.8$, $f_{Q,sep} = 1.22$, $E_{el,sep} = 4000$ GWh, $E_{Q,sep} = 300$ GWh.

| Allocation method | Expression for $f_{Q,chp}$ | Expression for $f_{el,chp}$ | Equation numbers | Case study $f_{Q,chp}$ | Case study $f_{el,chp}$ |
|---|---|---|---|---|---|
| IECR | $\frac{f_{F,chp} E_{F,chp} - f_{el,sep} E_{el,chp}}{E_{Q,chp}}$ | $f_{el,sep}$ | (4), (5) | 0.37 | 2.80 |
| IHRC | $f_{Q,sep}$ | $\frac{f_{F,chp} E_{F,chp} - f_{Q,sep} E_{Q,chp}}{E_{el,chp}}$ | (6), (7) | 1.22 | 1.94 |
| SPR | $\frac{f_{F,chp} E_{F,chp}}{f_{Q,sep} E_{Q,chp} + f_{el,sep} E_{el,chp}} f_{Q,sep}$ | $\frac{f_{F,chp} E_{F,chp}}{f_{Q,sep} E_{Q,chp} + f_{el,sep} E_{el,chp}} f_{el,sep}$ | (10) | 0.96 | 2.20 |
| STALPR | $\frac{(\sigma_{chp} + 1) f_{F,chp} \Phi_{loc}}{(\sigma_{chp} + \Phi_{loc}) \eta_{chp}}$ | $\frac{(\sigma_{chp} + 1) f_{F,chp}}{(\sigma_{chp} + \Phi_{loc}) \eta_{chp}}$ | (25) | 0.86 | 2.31 |
| Exergy | $\frac{f_{F,chp} E_{F,chp}}{\left(1 - \frac{T_{env}}{T_Q}\right) E_{Q,chp} + E_{el,chp}} \left(1 - \frac{T_{env}}{T_Q}\right)$ | $\frac{f_{F,chp} E_{F,chp}}{\left(1 - \frac{T_{env}}{T_Q}\right) E_{Q,chp} + E_{el,chp}}$ | (14) | 0.60 | 2.57 |

$$f^{IECR}_{Q,chp} = \frac{f_{F,chp}E_{F,chp} - f_{el,sep}E_{el,chp}}{E_{Q,chp}} \quad (5)$$

It is clear from Eq. (4) that the primary energy consumption attributed to the production of the cogenerated electricity, $f^{IECR}_{el,chp}E_{el,chp}$, is the primary energy that would be required to produce the same amount of electricity in a separate production facility, $f_{el,sep}E_{el,chp}$, and from Eq. (5) that the primary energy consumption attributed to the production of the cogenerated heat, $f^{IECR}_{Q,chp}E_{Q,chp}$, is the difference between the total primary energy consumption of the facility, $f_{F,chp}E_{F,chp}$, and the primary energy consumed for the separate production of the cogenerated electrical energy, $f_{el,sep}E_{el,chp}$. This electricity-centered method has sometimes been used in the early stages of district heating developments, but it is obsolete and unfair because it assigns the entire cogeneration savings benefit to the production of heat.

### 2.2. Incremental heat-centered reference (IHCR)

According to this (obsolete) method, Eq. (3) is set as

$$f^{IHCR}_{Q,chp} = f_{Q,sep} \quad (6)$$

so that by combining Eqs. (1) and (6) we obtain

$$f^{IHCR}_{el,chp} = \frac{f_{F,chp}E_{F,chp} - f_{Q,sep}E_{Q,chp}}{E_{el,chp}} \quad (7)$$

It is clear from Eq. (6) that the primary energy consumption attributed to the production of the cogenerated heat, $f^{IHCR}_{Q,chp}E_{Q,chp}$, is the primary energy that would be required to produce the same amount of heat in a separate production facility, $f_{Q,sep}E_{Q,chp}$, and from Eq. (7) that the primary energy consumption attributed to the production of the cogenerated electricity, $f^{IHCR}_{el,chp}E_{el,chp}$, is the difference between the total primary energy consumption of the facility, $f_{F,chp}E_{F,chp}$, and the primary energy consumed for the separate production of the cogenerated heat, $f_{Q,sep}E_{Q,chp}$. This heat-centered method has sometimes been used in the early stages of industrial cogeneration for waste heat, but it is obsolete and unfair because it assigns the entire cogeneration savings benefit to the production of electricity.

### 2.3. Separate productions reference (SPR)

According to this method Eq. (3) is set as

$$\frac{f^{SPR}_{Q,chp}}{f^{SPR}_{el,chp}} = \frac{f_{Q,sep}}{f_{el,sep}} \quad (8)$$

where $f_{el,sep}$ and $f_{Q,sep}$ are reference primary energy factors for the separate productions of electricity and heat, respectively. On account of Eqs. (1) and (2), Eq. (8) is equivalent to setting

$$\alpha^{SPR}_{el,chp} = \frac{f_{el,sep}E_{el,chp}}{f_{Q,sep}E_{Q,chp} + f_{el,sep}E_{el,chp}} \text{ and}$$
$$\alpha^{SPR}_{Q,chp} = \frac{f_{Q,sep}E_{Q,chp}}{f_{Q,sep}E_{Q,chp} + f_{el,sep}E_{el,chp}} \quad (9)$$

meaning that the primary energy consumption of cogenerated electricity and heat are both allocated based on the relative proportions of primary fuel consumption they would require in separate production facilities operating with the reference primary energy factors $f_{el,sep}$ and $f_{Q,sep}$, respectively. Combining Eqs. (2) and (9) yields the explicit expressions for the primary energy factors of the cogenerated electricity and heat, respectively,

$$f^{SPR}_{el,chp} = \frac{f_{F,chp}E_{F,chp}}{f_{Q,sep}E_{Q,chp} + f_{el,sep}E_{el,chp}} f_{el,sep} \text{ and}$$
$$f^{SPR}_{Q,chp} = \frac{f_{F,chp}E_{F,chp}}{f_{Q,sep}E_{Q,chp} + f_{el,sep}E_{el,chp}} f_{Q,sep} \quad (10)$$

This separate-production-centered method is the one currently preferred in most regulatory contexts.

### 2.4. Critique of the prevailing methods

The problem with the IECR method is that it assigns the entire benefit of cogeneration to the production of heat, thus making it appear that the cogenerated heat production has a very little primary energy factor. Let us consider a typical situation of public utilities of a city according to Example A.1 of Annex A of EN 15316-4-5:2007 [5]. The yearly consumption (based on lower heating value) of natural gas is 1000 GWh ($f_{F,chp} = 1.1$ for natural gas), the net heat production is 350 GWh, the net power production is 347 GWh, and it is assumed that $f_{el,sep} = 2.8$. Thus, using Eq. (5) according to the prescription in [5] yields $f^{IECR}_{Q,chp} = (1.1 \times 1000 - 2.8 \times 347)/350 = 0.37$, an unfairly low value that makes it very hard for all other heat production technologies to compete, and would discourage home owners with access to district heating to invest on energy-saving improvements.

Similarly, the IHCR method assigns the entire benefit of cogeneration to the production of electricity. For the same example Eq. (7) yields $f^{IHRC}_{el,chp} = (1.1 \times 1000 - 1.22 \times 350)/347 = 1.94$, again an unfairly low value.

The SPR method partly resolves these problems by providing more realistic figures; indeed, assuming $f_{Q,sep} = 1.1/0.9 = 1.22$ (where 0.9 is assumed as the reference efficiency of the typical separate production heat plant), Eqs. (10) yield the more realistic values $f^{SPR}_{Q,chp} = 1.1 \times 1000/(1.22 \times 350 + 2.8 \times 347) \times 1.22 = 0.96$ and $f^{SPR}_{el,chp} = 1.1 \times 1000/(1.22 \times 350 + 2.8 \times 347) \times 2.8 = 2.20$.

The SPR method has been proposed by various researchers not only in the context of evaluating fuel and greenhouse emission savings in combined heat and power facilities [5,8–10] but also in the combined production of power and desalinated water [22–25] which is important in several countries.

However, even the SPR method has a drawback which becomes increasingly important as the fraction of primary energy used in cogeneration facilities increases in a given local scenario. The problem is that the allocation within the cogeneration facility is based on some prescribed reference primary energy factors for the separate productions, $f_{Q,sep}$ and $f_{el,sep}$, fixed by some local authority and updated from time to time (2.80 for electricity and 1.22 for heat, in the examples above). These prescribed reference values in general differ from the average primary energy factors of electricity and heat in the considered local area. In other words, the SPR method, allocates the fuel consumption of the cogeneration plant on the basis of a hypothetical and less favorable scenario in which all the electricity and heat are produced separately with $f_{Q,sep}$ and $f_{el,sep}$.

This scenario becomes increasingly unrealistic as cogeneration gains more significant fractions of the energy market locally. For example, consider a scenario in which, in addition to the cogenerated 350 GWh of heat and 347 GWh of electricity, the local area has a yearly consumption of 300 GWh of separately produced heat with a primary energy factor $f_{Q,sep} = 1.22$ and 4000 GWh of separately produced electricity with a primary energy factor $f_{el,sep} = 2.80$. Then, with the SPR method applied to the cogeneration plant as above, the local average primary energy factor for electricity would be $f_{el,loc} = (2.2 \times 347 + 2.8 \times 4000)/4347 = 2.75$ and that for heat $f_{Q,loc} = (0.96 \times 350 + 1.22 \times 300)/650 = 1.08$

yielding a ratio $f_{Q,loc}/f_{el,loc} = 0.392$ which is quite different from the ratio $f_{Q,sep}/f_{el,sep} = 0.435$ which characterizes the separate productions-only scenario assumed as a reference by the SPR method.

In other words, this somewhat subtle inconsistency of the SPR method stems from the fact that it allocates fuel consumption by the cogeneration plant not on the fair basis of the locally prevailing actual average primary energy factors of electricity and heat, but on the basis of the hypothetical separate productions scenario which becomes increasingly unrealistic and distant from the actual local energy consumption scenario as cogeneration gains more significant fractions of the energy market. In our example, we could refine the allocation for the cogeneration facility by using the resulting local average primary energy factors 2.75 and 1.08, instead of 2.8 and 1.22, respectively. That would yield the primary energy factors for the cogenerated heat and electricity, respectively, $f^{SPR\prime}_{Q,chp} = 1.1 \times 1000/(1.08 \times 350 + 2.75 \times 347) \times 1.08 = 0.89$ and $f^{SPR\prime}_{el,chp} = 1.1 \times 1000/(1.08 \times 350 + 2.75 \times 347) \times 2.75 = 2.27$ showing that in the assumed local scenario, the original factors 0.96 and 2.20 credit electricity with a bit too much of the cogeneration benefit.

But this refinement is still somewhat inconsistent, in that the new values affect the local average primary energy factors, which become $f'_{el,loc} = (2.27 \times 347 + 2.8 \times 4000)/4347 = 2.76$ and $f'_{Q,loc} = (0.89 \times 350 + 1.22 \times 300)/650 = 1.04$, thus requiring additional iterations to obtain further refinements. The method we propose in Section 3 resolves this problem by keeping a logic similar to the SPR method, but substituting the static and hypothetical ratio $f_{Q,sep}/f_{el,sep}$ in the rhs of Eq. (8) with the dynamic ratio $f_{Q,loc}/f_{el,loc}$ which characterizes the actual local scenario.

In view of the key role of the local average primary energy factors $f_{Q,loc}$ and $f_{el,loc}$ in the proposed method, we provide in Section 3 their explicit formal definitions.

### 2.5. Exergy-based allocation method

According to this method [1–3,9,17], the allocation fractions are to be set as follows

$$\alpha_{el,chp} = \frac{Ex_{el,chp}}{Ex_{el,chp} + Ex_{Q,chp}} \text{ and } \alpha_{Q,chp} = \frac{Ex_{Q,chp}}{Ex_{el,chp} + Ex_{Q,chp}} \tag{11}$$

and is equivalent to setting Eq. (3) to

$$\frac{f^{Ex}_{Q,chp}}{f^{Ex}_{el,chp}} = \frac{Ex_{Q,chp}/E_{Q,chp}}{Ex_{el,chp}/E_{el,chp}} = 1 - \frac{T_{env}}{T_Q} \tag{12}$$

where $T_{env}$ is the temperature of the environment,

$$T_Q = \frac{h_{feed} - h_{return}}{s_{feed} - s_{return}} \tag{13}$$

is the equivalent single-heat-source delivery temperature, $h_{feed}$, $s_{feed}$ and $h_{return}$, $s_{return}$ are respectively the enthalpy and entropy of the feed and return streams with which the cogeneration facility delivers the thermal energy, and in writing Eqs. (12) and (13) we have taken into account that for electricity exergy and energy are the same, i.e., $Ex_{el,chp} = E_{el,chp}$, and for the heat $Ex_{Q,chp} = \dot{m}[h_{feed} - h_{return} - T_{env}(s_{feed} - s_{return})]$ and $E_{Q,chp} = \dot{m}[h_{feed} - h_{return}]$. Clearly, for heat delivered in the form of heating a gas or liquid stream with negligible pressure drop and constant specific heat capacity, $T_Q$ is the log-mean temperature $T_Q = (T_{feed} - T_{return})/\ln(T_{feed}/T_{return})$.

Combining Eqs. (1), (12) and (13) yields the explicit expressions for the primary energy factors of the cogenerated electricity and heat

$$f^{Ex}_{el,chp} = \frac{f_{F,chp}E_{F,chp}}{\left(1 - \frac{T_{env}}{T_Q}\right)E_{Q,chp} + E_{el,chp}} \text{ and}$$
$$f^{Ex}_{Q,chp} = \frac{f_{F,chp}E_{F,chp}}{\left(1 - \frac{T_{env}}{T_Q}\right)E_{Q,chp} + E_{el,chp}}\left(1 - \frac{T_{env}}{T_Q}\right) \tag{14}$$

This method amounts to allocating the primary energy consumption of cogenerated electricity and heat based on the relative proportions of the primary fuel consumptions they would require in a hypothetic scenario in which all machinery operates with the highest possible production and conversion efficiencies compatible with thermodynamic limitations, i.e., reversibly.

For our example, assuming the heat is delivered via pressurized water with a feed temperature of $T_{feed} = 120$ °C and a return temperature of $T_{return} = 60$ °C, as typical of winter-time operation in district heating when the environmental temperature is $T_{env} = 5$ °C, the log-mean delivery temperature is $T_Q = 89.2$ °C. Substituting in Eqs. (14) yield for the primary energy factor of cogenerated electricity, $f^{Ex}_{el,chp} = 2.57$ and for the primary energy factor of cogenerated heat, $f^{Ex}_{Q,chp} = 0.597$.

When compared with the corresponding SPR values, 2.20 and 0.96, respectively, as well as with the STALPR values (see next section), 2.31 and 0.86, respectively, these figures show that the exergy-based method credits the thermal energy with too high a share of the cogeneration benefit leaving an unfairly little share of the fuel savings to the cogenerated electricity. This is because currently the average second-law efficiencies of power production are much closer to 100% (the reversible-scenario value taken as reference for both electricity and heat according to this method) than the average second-law efficiencies of heat production. In other words, though based on sound thermodynamic reasoning, the exergy-based method assumes as reference hypothetical efficiencies that are too distant from the average efficiencies of the current industrial and technological scenario. Therefore, if adopted as a basis of regulations, this method would result in market distortions which may for example give too much advantage to district heating systems thus improperly discouraging home owners that have access to district heating from investing in energy-saving improvements such as better building and window insulation and the like.

## 3. Rationale of the proposed method

In this section we present the proposed method in the particular case of the energy generation scenario shown in Fig. 1 with *n* electricity plants, *m* heat plants, and a single cogeneration facility, pursuing for comparison the same example considered in the previous section, and focusing on determining the primary energy factors of cogenerated electricity and heat. The subsequent sections generalize the method to multiple cogeneration facilities and a multiple-products scenario.

The basic rationale of the proposed method is that the allocation parameters to be used to assign primary energy factors to cogenerated electricity and heat should not be static reference values fixed by some authority and updated from time to time, but should be self-determined by the method itself as characteristic average features of the actual energy production scenario of electricity and heat delivery to the given local area of interest. For this reason we call it the Self-Tuned Average-Local-Productions Reference (STALPR) method.

First we define $f_{el,loc}$ and $f_{Q,loc}$ as the average primary energy factors of electricity and heat produced by the plants that serve the local area of interest. With reference to Fig. 1, they are calculated as

$$f_{el,loc} = \frac{\sum_{i=1}^{n} f_{el,sep,i}E_{el,sep,i} + f_{el,chp}E_{el,chp}}{\sum_{i=1}^{n} E_{el,sep,i} + E_{el,chp}} \text{ and}$$
$$f_{Q,loc} = \frac{\sum_{i=1}^{m} f_{Q,sep,i}E_{Q,sep,i} + f_{Q,chp}E_{Q,chp}}{\sum_{i=1}^{m} E_{Q,sep,i} + E_{Q,chp}} \tag{15}$$

Then, we follow a logic similar to that adopted within the classical SPR method to provide a closure to the system of Eqs. (1) and (3), but instead of Eq. (3) we adopt the following closure rule to determine the primary energy allocation,

$$\frac{f_{Q,chp}}{f_{el,chp}} = \frac{f_{Q,loc}}{f_{el,loc}} \tag{16}$$

or, equivalently,

$$\alpha_{Q,chp}^{STALPR} = \frac{f_{Q,loc}E_{Q,chp}}{f_{el,loc}E_{el,chp} + f_{Q,loc}E_{Q,chp}} \text{ and}$$
$$\alpha_{el,chp}^{STALPR} = \frac{f_{el,loc}E_{el,chp}}{f_{el,loc}E_{el,chp} + f_{Q,loc}E_{Q,chp}} \tag{17}$$

meaning that the primary energy consumption of cogenerated electricity and heat are both allocated based on the relative proportions of the actual average primary energy consumption they require in the local area, which includes that of the cogeneration facility itself.

Combining Eqs. (2) and (17) yields the explicit expressions for the primary energy factors of the cogenerated electricity and heat, respectively,

$$f_{el,chp}^{STALPR} = \frac{f_{F,chp}E_{F,chp}}{f_{Q,loc}E_{Q,chp} + f_{el,loc}E_{el,chp}} f_{el,loc} \text{ and}$$
$$f_{Q,chp}^{STALPR} = \frac{f_{F,chp}E_{F,chp}}{f_{Q,loc}E_{Q,chp} + f_{el,loc}E_{el,chp}} f_{Q,loc} \tag{18}$$

It is noteworthy that the system of Eqs. (1), (15) and (16) is nonlinear in the four unknowns $f_{el,loc}$, $f_{Q,loc}$, $f_{el,chp}$, $f_{Q,chp}$. However, an analytical solution can be readily obtained as follows.

By defining the following ratios

$$\sigma_{chp} = \frac{E_{el,chp}}{E_{Q,chp}} \tag{19}$$

$$\eta_{chp} = \frac{E_{el,chp} + E_{Q,chp}}{E_{F,chp}} \tag{20}$$

$$\Phi_{loc} = \frac{f_{Q,loc}}{f_{el,loc}} \tag{21}$$

$$\sigma_{loc} = \frac{\sum_{i=1}^{n} E_{el,sep,i} + E_{el,chp}}{\sum_{i=1}^{m} E_{Q,sep,i} + E_{Q,chp}} \tag{22}$$

the allocation fractions and the primary energy factors given by Eqs. (17) and (18) can be written as follows (we omit the superscript STALPR for simplicity of notation).

$$\alpha_{Q,chp} = \frac{\Phi_{loc}}{\sigma_{chp} + \Phi_{loc}} \text{ and } \alpha_{el,chp} = \frac{\sigma_{chp}}{\sigma_{chp} + \Phi_{loc}} \tag{23}$$

$$f_{Q,chp} = \frac{\alpha_{Q,chp} f_{F,chp} E_{F,chp}}{E_{Q,chp}} \text{ and } f_{el,chp} = \frac{\alpha_{el,chp} f_{F,chp} E_{F,chp}}{E_{el,chp}} \tag{24}$$

or, equivalently,

$$f_{Q,chp} = \frac{\left(\sigma_{chp}+1\right) f_{F,chp}\Phi_{loc}}{\left(\sigma_{chp}+\Phi_{loc}\right)\eta_{chp}} \text{ and } f_{el,chp} = \frac{\left(\sigma_{chp}+1\right) f_{F,chp}}{\left(\sigma_{chp}+\Phi_{loc}\right)\eta_{chp}} \tag{25}$$

By defining the fractions of cogenerated electricity and heat delivered to the local area and the average primary energy factors of the separate productions, respectively,

$$\gamma_{Q,chp} = 1 - \gamma_{Q,sep} = \frac{E_{Q,chp}}{\sum_{i=1}^{m} E_{Q,sep,i} + E_{Q,chp}} \text{ and}$$
$$\gamma_{el,chp} = 1 - \gamma_{el,sep} = \frac{E_{el,chp}}{\sum_{i=1}^{n} E_{el,sep,i} + E_{el,chp}} = \frac{\sigma_{chp}\gamma_{Q,chp}}{\sigma_{loc}} \tag{26}$$

$$\bar{f}_{Q,sep} = \frac{\sum_{i=1}^{m} f_{Q,sep,i}E_{Q,sep,i}}{\sum_{i=1}^{m} E_{Q,sep,i}} \text{ and } \bar{f}_{el,sep} = \frac{\sum_{i=1}^{n} f_{el,sep,i}E_{el,sep,i}}{\sum_{i=1}^{n} E_{el,sep,i}} \tag{27}$$

the average primary energy factors may be written as

$$f_{Q,loc} = \left(1-\gamma_{Q,chp}\right)\bar{f}_{Q,sep} + \gamma_{Q,chp} f_{Q,chp} \text{ and}$$
$$f_{el,loc} = \left(1-\gamma_{el,chp}\right)\bar{f}_{el,sep} + \gamma_{el,chp} f_{el,chp} \tag{28}$$

It is noteworthy that, combining Eqs. (16), (26) and (28), we have the relation

$$\left(1-\gamma_{Q,chp}\right)\frac{\bar{f}_{Q,sep}}{f_{Q,chp}} + 1 - \frac{\sigma_{chp}}{\sigma_{loc}} = \left(1-\gamma_{Q,chp}\frac{\sigma_{chp}}{\sigma_{loc}}\right)\frac{\bar{f}_{el,sep}}{f_{el,chp}}$$

which implies, for example, that

$$f_{el,chp} = \bar{f}_{el,sep} \text{ whenever } \gamma_{Q,chp} = 1$$

$$f_{Q,chp}/\bar{f}_{Q,sep} = \sigma_{loc}/\sigma_{chp} \text{ whenever } \gamma_{Q,chp} = \sigma_{loc}/\sigma_{chp}$$

Taking the ratio of the two Eqs. (28) to compute $\Phi_{loc}$ according to Eq. (21) and using Eqs. (25) to eliminate $f_{el,chp}$, and $f_{Q,chp}$, we obtain the following relation

$$\Phi_{loc} = \frac{\left(1-\gamma_{Q,chp}\right)\bar{f}_{Q,sep} + \gamma_{Q,chp}\dfrac{\left(\sigma_{chp}+1\right) f_{F,chp}\Phi_{loc}}{\left(\sigma_{chp}+\Phi_{loc}\right)\eta_{chp}}}{\left(1-\gamma_{el,chp}\right)\bar{f}_{el,sep} + \gamma_{el,chp}\dfrac{\left(\sigma_{chp}+1\right) f_{F,chp}}{\left(\sigma_{chp}+\Phi_{loc}\right)\eta_{chp}}} \tag{29}$$

which clearly defines $\Phi_{loc}$ implicitly in terms of the parameters $\sigma_{chp}$, $\eta_{chp}$, $f_{F,chp}$ of the cogeneration plant and the local parameters $\gamma_{el,chp}$, $\gamma_{Q,chp}$, $\bar{f}_{el,sep}$ and $\bar{f}_{Q,sep}$.

With a few rearrangements and using the last of Eqs. (26), Eq. (29) can be finally cast as follows

$$\begin{aligned}
&\left(\sigma_{loc} - \gamma_{Q,chp}\sigma_{chp}\right)\eta_{chp}\bar{f}_{el,sep}\Phi_{loc}^2 + \Big[f_{F,chp}\gamma_{Q,chp}\left(\sigma_{chp}-\sigma_{loc}\right) \\
&\times\left(\sigma_{chp}+1\right) + \left(\sigma_{loc}-\gamma_{Q,chp}\sigma_{chp}\right)\sigma_{chp}\eta_{chp}\bar{f}_{el,sep} \\
&-\left(1-\gamma_{Q,chp}\right)\sigma_{loc}\eta_{chp}\bar{f}_{Q,sep}\Big]\Phi_{loc} \\
&-\left(1-\gamma_{Q,chp}\right)\sigma_{chp}\sigma_{loc}\eta_{chp}\bar{f}_{Q,sep} = 0
\end{aligned} \tag{30}$$

This second order equation in $\Phi_{loc}$ can be easily solved for the only positive root it admits (see the Appendix). Once $\Phi_{loc}$ is found, the primary energy factors $f_{el,chp}$ and $f_{Q,chp}$ can be obtained from Eqs. (25) and the values of $f_{el,loc}$ and $f_{Q,loc}$ from Eqs. (28).

A detailed analysis of the dependence of $\Phi_{loc}$ on the various parameters of the local area is reported in the Appendix. In particular, it is important to study the dependence of $\Phi_{loc}$ on $\gamma_{Q,chp}$ because it defines how $f_{el,chp}$ and $f_{Q,chp}$ change with the penetration of cogeneration (represented in this particular case by the size of the single chp plant shown in Fig. 1). In fact, according to Eqs. (25) an increase in $\Phi_{loc}$ always implies an increase in $f_{Q,chp}$ and a corresponding reduction in $f_{el,chp}$, and viceversa.[1]

For our example, the parameters are: $\sigma_{chp} = 347/350 = 0.9914$, $\eta_{chp} = (347 + 350)/1000 = 0.697$, $f_{F,chp} = 1.1$, $\sigma_{loc} = (4000 + 347)/(300 + 350) = 6.69$, $\gamma_{el,sep} = 4000/(4000 + 347) = 0.92$, $\gamma_{Q,sep} = 300/(300 + 350) = 0.46$, $\gamma_{Q,chp} = 350/(300 + 350) = 0.54$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.22$. With these parameters, the single nonnegative root of Eq. (30) is $\Phi_{loc} = 0.371$ and the results are: $\alpha_{el,chp} = 0.728$, $\alpha_{Q,chp} = 0.272$, $f_{el,chp} = 2.31$, $f_{Q,chp} = 0.86$, $f_{el,loc} = 2.75$, $f_{Q,loc} = 0.977$. Table 1 summarizes the relevant expressions for the heat and electricity primary energy factors according to the various allocation methods (the last two columns summarize the values obtained in the examples carried out in Sections 2 and 3).

### 3.1. General formulation for multiple heat and power cogeneration facilities in the local area

The STALPR method in the general case of a local area scenario with $r$ heat and power cogeneration facilities is formulated as follows, by simply generalizing the single-facility formulation just outlined. The average primary energy factors of electricity and heat produced by the plants that serve the local area are calculated as

$$f_{el,loc} = \frac{\sum_{i=1}^{n} f_{el,sep,i}E_{el,sep,i} + \sum_{i=1}^{r} f_{el,chp,i}E_{el,chp,i}}{\sum_{i=1}^{n} E_{el,sep,i} + \sum_{i=1}^{r} E_{el,chp,i}} \text{ and}$$
$$f_{Q,loc} = \frac{\sum_{i=1}^{m} f_{Q,sep,i}E_{Q,sep,i} + \sum_{i=1}^{r} f_{Q,chp,i}E_{Q,chp,i}}{\sum_{i=1}^{m} E_{Q,sep,i} + \sum_{i=1}^{r} E_{Q,chp,i}} \quad (31)$$

The primary energy balance for each CHP facility is

$$f_{F,chp,i}E_{F,chp,i} = f_{Q,chp,i}E_{Q,chp,i} + f_{el,chp,i}E_{el,chp,i} \quad (32)$$

and the definition of the allocation fractions are

$$\alpha_{Q,chp,i} = \frac{f_{Q,chp,i}E_{Q,chp,i}}{f_{F,chp,i}E_{F,chp,i}} \text{ and } \alpha_{el,chp,i} = \frac{f_{el,chp,i}E_{el,chp,i}}{f_{F,chp,i}E_{F,chp,i}} \quad (33)$$

We adopt the following closure rule to determine the fair primary energy allocation,

$$\frac{f_{Q,chp,i}}{f_{el,chp,i}} = \frac{f_{Q,loc}}{f_{el,loc}} \text{ for every } i = 1, 2, ..., r \quad (34)$$

or, equivalently,

$$\alpha^{SDALPR}_{Q,chp,i} = \frac{f_{Q,loc}E_{Q,chp,i}}{f_{el,loc}E_{el,chp,i} + f_{Q,loc}E_{Q,chp,i}} \text{ and}$$
$$\alpha^{SDALPR}_{el,chp,i} = \frac{f_{el,loc}E_{el,chp,i}}{f_{el,loc}E_{el,chp,i} + f_{Q,loc}E_{Q,chp,i}} \quad (35)$$

[1] The partial derivatives of the two Eqs. (25) $\frac{\partial f_{Q,chp}}{\partial \Phi_{loc}} = \frac{(\sigma_{chp}+1)f_{F,chp}\sigma_{chp}}{(\sigma_{chp}+\Phi_{loc})^2\eta_{chp}}$ and $\frac{\partial f_{el,chp}}{\partial \Phi_{loc}} = -\frac{(\sigma_{chp}+1)f_{F,chp}}{(\sigma_{chp}+\Phi_{loc})^2\eta_{chp}}$ are indeed always positive and negative, respectively.

meaning again that the primary energy consumption of cogenerated electricity and heat in each facility are both allocated based on the relative proportions of the actual average primary energy consumption they require in the local area, taking into account all the locally existing cogeneration facilities.

Combining Eqs. (33) and (35) yields the explicit expressions for the primary energy factors of the cogenerated electricity and heat, respectively,

$$f^{SDALPR}_{el,chp,i} = \frac{f_{F,chp,i}E_{F,chp,i}}{f_{Q,loc}E_{Q,chp,i} + f_{el,loc}E_{el,chp,i}} f_{el,loc} \text{ and}$$
$$f^{SDALPR}_{Q,chp,i} = \frac{f_{F,chp,i}E_{F,chp,i}}{f_{Q,loc}E_{Q,chp,i} + f_{el,loc}E_{el,chp,i}} f_{Q,loc} \quad (36)$$

Again, we note that the system of Eqs. (31), (32) and (34) is non linear in the $r$+1 unknowns $f_{el,loc}$, $f_{Q,loc}$, $f_{el,chp,i}$, $f_{Q,chp,i}$. However, an analytical solution can be obtained as follows.

By defining the ratios

$$\sigma_{chp,i} = \frac{E_{el,chp,i}}{E_{Q,chp,i}} \quad (37)$$

$$\eta_{chp,i} = \frac{E_{el,chp,i} + E_{Q,chp,i}}{E_{F,chp,i}} \quad (38)$$

$$\Phi_{loc} = \frac{f_{Q,loc}}{f_{el,loc}} \quad (39)$$

the allocation fractions and the primary energy factors can be written as follows

$$\alpha_{el,chp,i} = \frac{\sigma_{chp,i}}{\sigma_{chp,i} + \Phi_{loc}} \text{ and } \alpha_{Q,chp,i} = \frac{\Phi_{loc}}{\sigma_{chp,i} + \Phi_{loc}} \quad (40)$$

$$f_{el,chp,i} = \frac{\left(\sigma_{chp,i} + 1\right)f_{F,chp,i}}{\left(\sigma_{chp,i} + \Phi_{loc}\right)\eta_{chp,i}} \text{ and}$$
$$f_{Q,chp,i} = \frac{\left(\sigma_{chp,i} + 1\right)f_{F,chp,i}\Phi_{loc}}{\left(\sigma_{chp,i} + \Phi_{loc}\right)\eta_{chp,i}} \quad (41)$$

The fractions of separately produced electricity and heat delivered in the local area and the average primary energy factors may be written, respectively, as

$$\gamma_{el,sep} = \frac{\sum_{i=1}^{n} E_{el,sep,i}}{\sum_{i=1}^{n} E_{el,sep,i} + \sum_{i=1}^{r} E_{el,chp,i}} \text{ and}$$
$$\gamma_{Q,sep} = \frac{\sum_{i=1}^{m} E_{Q,sep,i}}{\sum_{i=1}^{m} E_{Q,sep,i} + \sum_{i=1}^{r} E_{el,chp,i}} \quad (42)$$

The average primary energy factors may be written as

$$f_{el,loc} = \gamma_{el,sep}\bar{f}_{el,sep} + \left(1 - \gamma_{el,sep}\right)\bar{f}_{el,chp} \text{ and}$$
$$f_{Q,loc} = \gamma_{Q,sep}\bar{f}_{Q,sep} + \left(1 - \gamma_{Q,sep}\right)\bar{f}_{Q,chp} \quad (43)$$

where, in addition to Eqs. (27), we also defined

$$\bar{f}_{el,chp} = \frac{\sum_{i=1}^{r} f_{el,chp,i}E_{el,chp,i}}{\sum_{i=1}^{n} E_{el,chp,i}} = \sum_{i=1}^{r}\gamma_{el,chp,i}f_{el,chp,i} \text{ and}$$
$$\bar{f}_{Q,chp} = \frac{\sum_{i=1}^{r} f_{Q,chp,i}E_{Q,chp,i}}{\sum_{i=1}^{m} E_{Q,chp,i}} = \sum_{i=1}^{r}\gamma_{Q,chp,i}f_{Q,chp,i} \quad (44)$$

$$\gamma_{el,chp,j} = \frac{E_{el,chp,j}}{\sum_{i=1}^{r} E_{el,chp,i}} \text{ and } \gamma_{Q,chp,j} = \frac{E_{Q,chp,j}}{\sum_{i=1}^{r} E_{Q,chp,i}} \quad (45)$$

Taking the ratio of the two Eqs. (43) to compute $\Phi_{loc}$ and using Eqs. (41) and (44) to eliminate $\bar{f}_{el,chp}$ and $\bar{f}_{Q,chp}$,we obtain the following relation

$$\Phi_{loc} = \frac{\gamma_{Q,sep}\bar{f}_{Q,sep} + \left(1-\gamma_{Q,sep}\right) \sum_{i=1}^{r} \gamma_{Q,chp,i} \dfrac{\left(\sigma_{chp,i}+1\right) f_{F,chp,i}\Phi_{loc}}{\left(\sigma_{chp,i}+\Phi_{loc}\right)\eta_{chp,i}}}{\gamma_{el,sep}\bar{f}_{el,sep} + \left(1-\gamma_{el,sep}\right) \sum_{i=1}^{r} \gamma_{el,chp,i} \dfrac{\left(\sigma_{chp,i}+1\right) f_{F,chp,i}}{\left(\sigma_{chp,i}+\Phi_{loc}\right)\eta_{chp,i}}} \quad (46)$$

which again defines $\Phi_{loc}$ implicitly in terms of the parameters $\sigma_{chp,i}$, $\eta_{chp,i}$, $f_{F,chp,i}$, $\gamma_{el,chp,i}$, and $\gamma_{Q,chp,i}$ of the cogeneration facilities and the local parameters $\gamma_{el,sep}$, $\gamma_{Q,sep}$, $\bar{f}_{el,sep}$ and $\bar{f}_{Q,sep}$. This equation can in principle be cast as an $r+1$ order equation in $\Phi_{loc}$, however, in practice its roots must be found numerically and among them we must pick out the positive one which, when substituted in Eq. (38), does not yield any negative $f_{el,chp,i}$ or $f_{Q,chp,i}$.

For example, consider the area of our previous example to which we add a second CHP plant producing 150 GWh of electrical energy and 250 GWh of heat and consuming 500 GWh of the same fuel. The parameters are, therefore: $\sigma_{chp,1} = 347/350 = 0.9914$, $\eta_{chp,1} = (347 + 350)/1000 = 0.697$, $\sigma_{chp,2} = 150/250 = 0.60$, $\eta_{chp,2} = (150 + 250)/500 = 0.80$, $f_{F,chp,1} = f_{F,chp,2} = 1.1$, $\gamma_{el,chp,1} = 1-\gamma_{el,chp,2} = 347/(347 + 150) = 0.698$, $\gamma_{Q,chp,1} = 1-\gamma_{Q,chp,2} = 350/(350+250) = 0.583$, $\gamma_{el,sep} = 4000/(4000+347+150) = 0.889$, $\gamma_{Q,sep} = 300/(300+350+250) = 0.333$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.22$. With these parameters, Eq. (46) has the single nonnegative root $\Phi_{loc} = 0.343$ and the results are: $f_{el,chp,1} = 2.36$, $f_{el,chp,2} = 2.33$, $\bar{f}_{el,chp} = 2.35$, $f_{Q,chp,1} = 0.808$, $f_{Q,chp,2} = 0.800$, $\bar{f}_{Q,chp} = 0.804$, $f_{el,loc} = 2.75$, $f_{Q,loc} = 0.94$.

## 4. Comparison of the allocation methods

In order to better focus on the features of the STALPR approach, a comparison with the classical methods is carried out on the basis of following example. We consider that the local area shown in Fig. 1 is representative of a generic mix of industrial, residential and tertiary activities so that the overall yearly thermal energy consumption is twice the consumption of electricity, thus resulting in $\sigma_{loc} = 0.5$. We assume that heat and electricity are initially produced by means of a certain number of separate production plants, considered for simplicity identical to one another and characterized by electricity and heat primary energy factors $f_{el,sep,i} = \bar{f}_{el,sep} = 2.4$ and $f_{Q,sep,i} = \bar{f}_{Q,sep} = 1.22$. Then we consider that the separately produced heat and electricity are progressively replaced by cogeneration plants. To select realistic values for the parameters of the example, we identify two distinct cases on the basis of which the following typical chp technology is implemented:

a) steam cycle with back-pressure steam turbine (BPST) operating with average $\eta_{chp} = 85\%$ and $\sigma_{chp} = 0.2$;
b) combined cycle (CC) operating with average $\eta_{chp} = 78\%$ and $\sigma_{chp} = 1.2$.

In either case, each chp plant is assumed identical to the others so that the "*r*" installations can be treated as a single unit of equivalent size, therefore allowing the straightforward solution of Eq. (30). The degree of penetration of heat cogeneration in the local area, defined by the size of the equivalent chp plant, is then measured by the value of the parameter $\gamma_{Q,chp}$. It is noteworthy that in case of CC, where $\sigma_{chp} > \sigma_{loc}$, the condition $0 < \gamma_{Q,chp} < \min(1,\sigma_{chp}/\sigma_{loc})$ [or the equivalent expression $0 \le x \le \min(1,1/1-a)$ in terms of the alternative set of variables defined in the Appendix] limits the penetration of heat cogeneration to the maximum value $\gamma_{Q,chp}^{max} = 0.417$ reached when all the electricity demand of the local area is cogenerated, $\gamma_{el,chp} = 1$. In case of BPST, where $\sigma_{chp} < \sigma_{loc}$, the limiting value is $\gamma_{Q,chp}^{max} = 1$, which refers to the situation when all the heat is produced by cogeneration. Table 2 summarizes the assumptions considered in this example. Fig. 2 shows the results of the analysis in terms of primary energy factors and $\Phi_{loc}$ profiles, plotted as a function of $\gamma_{Q,chp}$. The corresponding values given by the classical methods are given in Table 3.

It should be noted that according to the classical methods the primary energy factors do not change with $\gamma_{Q,chp}$ (Table 3) as they refer to the fixed scenario of standard separate productions with $f_{el,sep} = 2.4$ and $f_{Q,sep} = 1.22$. To the contrary, the resulting profiles of f's and $\Phi_{loc}$ (Fig. 2) provided by the STALPR method are interesting and reasonable functions of $\gamma_{Q,chp}$, in accordance with the detailed analysis of Eq. (30) carried out in the Appendix. In particular, in case of CC ($\sigma_{chp} > \sigma_{loc}$, i.e., $a < 0$ in the notation of the Appendix) $\Phi_{loc}$ increases with $\gamma_{Q,chp}$. As already observed this implies that $f_{Q,chp}$ and $f_{el,chp}$ respectively increase and decrease with $\gamma_{Q,chp}$, meaning that as heat cogeneration further penetrates in the local area, a higher fraction of primary energy of the chp plants is allocated to the production of heat. This is consistent with the fact that the chp plants have in this case a higher proportion of electricity than that required by the local area ($\sigma_{chp} > \sigma_{loc}$) and thus the penetration of heat cogeneration goes together with an even higher penetration of electricity cogeneration, which therefore takes up a higher share of the overall cogeneration benefits. Analogous considerations can be

**Table 2**
Assumptions for the case study illustrated in Fig. 2.

| Parameters of the local area | | |
|---|---|---|
| $\sigma_{loc}$ | 0.5 | |
| $f_{el,sep,i} = \bar{f}_{el,sep}$ | 2.4 | |
| $f_{Q,sep,i} = \bar{f}_{Q,sep}$ | 1.22 | |
| Parameters of the chp plants | | |
| Technology | CC | BPST |
| $f_{F,chp,i}$ | 1.1 | 1.1 |
| $\sigma_{chp}$ | 1.2 | 0.2 |
| $\eta_{chp,i}$ | 78% | 85% |
| Parameters in the notation of the Appendix | | |
| $a$ | −1.4 | 0.6 |
| $b$ | 0.243 | 0.087 |
| $c$ | 2.361 | 0.393 |
| $d$ | 0.508 | 0.508 |
| $\eta^*$ | 0.590 | 0.776 |

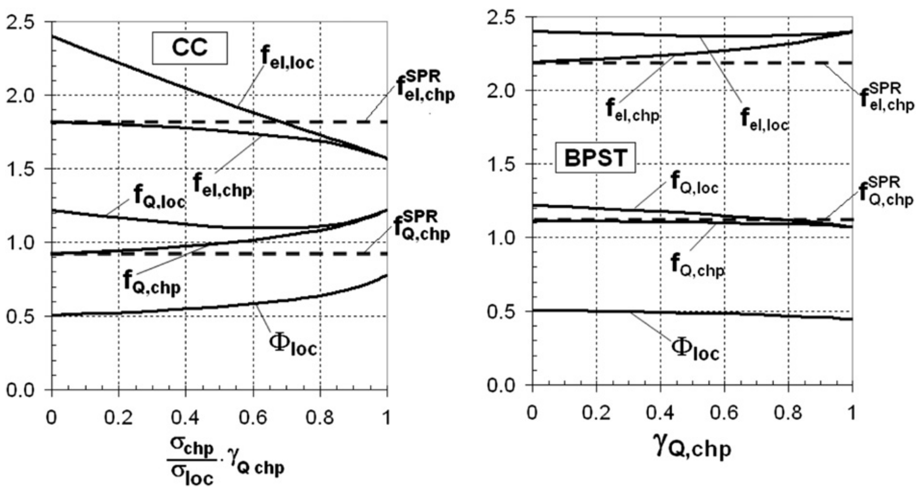


**Fig. 2.** $\Phi_{loc}$ and the primary energy factors plotted as functions of the parameter $\gamma_{Q,chp}$ for the values listed in Table 2. Left: CHP facilities based on combined cycle (CC) technology. Right: CHP facilities based on steam cycle technology with back-pressure steam turbine (BPST). Dashed lines in both figures refer to the primary energy factors calculated with the classical SPR method.

made for the case of BPST (where $\sigma_{chp} < \sigma_{loc}$), resulting in the opposite implication on $f_{Q,chp}$ and $f_{el,chp}$.

Furthermore it is possible to determine the values that the f's assume at the limits of the meaningful range of $\gamma_{Q,chp}$. At $\gamma_{Q,chp} = 0$ (no cogeneration in the local area), as shown in the Appendix, we have $\Phi_{loc} = \bar{f}_{Q,sep}/\bar{f}_{el,sep} = 0.508$. Substituting into Eqs. (25) we obtain $f_{Q,chp} = 0.923$ and $f_{el,chp} = 1.816$ in the CC case and $f_{Q,chp} = 1.114$ and $f_{el,chp} = 2.192$ in the BPST case. At $\gamma_{Q,chp} = \min(1,\sigma_{chp}/\sigma_{loc})$ (maximum penetration of heat cogeneration in the area), for the CC case, applying Eqs. (A4) we obtain $\Phi_{loc} = 0.778$, $f_{Q,chp} = 1.22$ and $f_{el,chp} = 1.569$, and for the BPST case, applying Eqs. (A5) we obtain $\Phi_{loc} = 0.447$, $f_{Q,chp} = 1.073$ and $f_{el,chp} = 2.4$.

Fig. 2 also shows that the local primary energy factors $f_{Q,loc}$ and $f_{el,loc}$ at $\gamma_{Q,chp} = 0$ coincide with the corresponding values of the separate production as obtained from substituting $\gamma_{Q,chp} = 0$ and $\gamma_{el,chp} = 0$ into Eqs. (24); notably at $\gamma_{Q,chp} = \min(1,\sigma_{chp}/\sigma_{loc})$ we have $f_{Q,loc} = f_{Q,chp}$ and $f_{el,loc} = f_{el,chp}$. This is obtained by substituting the limiting values $\gamma_{Q,chp} = 1$ (valid when $\sigma_{chp} < \sigma_{loc}$) and $\gamma_{el,chp} = 1$ (valid when $\sigma_{chp} > \sigma_{loc}$) respectively in the first and the second of Eqs. (28) and recalling that by definition $f_{Q,loc}/f_{el,loc} = f_{Q,chp}/f_{el,chp}$.

Moreover it is worth observing that with the increase of $\gamma_{Q,chp}$, the chp primary energy factors calculated with the STALPR method progressively depart, as expected, from the values obtained with the SPR method. In our case study, the maximum difference in $f_{Q,chp}$ between the two methods is 3.7% for the BPST case and 32.1% for the CC case, while the maximum difference in $f_{el,chp}$ is 9.5% for the BPST case and 13.6% for the CC case. Note also that the values of the separate production f's used in the STALPR method are not to be fixed by some authority but they are those that characterize the actual separate production situation in the local area, Eqs. (27).

Finally, it is worth noting that the cases illustrated by Fig. 2 are representative of the two main situations of practical interest, characterized by either the condition $\sigma_{chp} > \sigma_{loc}$ ($\Phi_{loc}$ increasing with $\gamma_{Q,chp}$) or $\sigma_{chp} < \sigma_{loc}$ ($\Phi_{loc}$ decreasing with $\gamma_{Q,chp}$). In general, any other possible combination of the parameters of the local area and CHP plant are included in one of these two cases (for the particular case $\sigma_{chp} = \sigma_{loc}$ and other particular cases, see the Appendix). The general rules reported in the Appendix provide a useful preliminary estimate of the heat and electricity primary energy factors resulting from inserting a CHP facility. To better explain this, let us consider for instance a local area where $\sigma_{loc} = 2$, meaning that the overall yearly thermal energy consumption is half the consumption of electricity, being the other parameters of the problem the same as in Table 2. According to the notation defined in the Appendix we obtain for the two power plant technologies the set of parameters shown in Table 4.

The following analysis can be formulated. Since in both cases we have $a > 0$, $\Phi_{loc}$ decreases with $\gamma_{Q,chp}$ (implying that $f_{Q,chp}$ and $f_{el,chp}$ respectively decrease and increase with $\gamma_{Q,chp}$) and maximum penetration of cogeneration is reached for $\gamma_{Q,chp} = 1$. Furthermore, in the BPST case: at $\gamma_{Q,chp} = 0$ (no cogeneration in the local area), $\Phi_{loc} = d = 0.508$ which through Eqs. (25) yields $f_{Q,chp} = 1.114$ and $f_{el,chp} = 2.192$; at $\gamma_{Q,chp} = 1$, since $a > 0$ and $(c+1)b < 1$, from Eqs. (A5) we obtain $\Phi_{loc} = 0.447$, $f_{Q,chp} = 1.073$, and $f_{el,chp} = 2.4$. Similarly, in the CC case: at $\gamma_{Q,chp} = 0$ we have $\Phi_{loc} = d = 0.508$ which through Eqs. (25) yields $f_{Q,chp} = 0.923$ and $f_{el,chp} = 1.816$. At $\gamma_{Q,chp} = 1$, since $a > 0$ and $(c+1)b < 1$ from Eqs. (A5) we obtain $\Phi_{loc} = 0.093$, $f_{Q,chp} = 0.223$, and $f_{el,chp} = 2.4$. The plots of $\Phi_{loc}$ and the primary energy factors as functions of the parameter $\gamma_{Q,chp}$ are shown in Fig. 3.

## 5. Generalization to multiple cogeneration of several different energy-intensive products

In Section 3 we have seen how context-dependent allocation of cogeneration benefits results in a self-consistent assignment of fair primary energy factors to heat and electric power. In the present section we generalize the STALPR method to a local area scenario

**Table 3**
Values of the heat and electricity primary energy factors calculated for the case studies of Figs. 2 and 3 by means of the classical allocation methods.

| CC | | | | | | BPST | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| IECR | | IHCR | | SPR | | IECR | | IHCR | | SPR | |
| $f_{el,chp}$ | $f_{Q,chp}$ | $f_{el,chp}$ | $f_{Q,chp}$ | $f_{el,chp}$ | $f_{Q,chp}$ | $f_{el,chp}$ | $f_{Q,chp}$ | $f_{el,chp}$ | $f_{Q,chp}$ | $f_{el,chp}$ | $f_{Q,chp}$ |
| 2.4 | 0.223 | 1.569 | 1.22 | 1.816 | 0.923 | 2.4 | 1.073 | 1.665 | 1.22 | 2.192 | 1.114 |

**Table 4**
Assumptions for the case study illustrated in Fig. 3.

| Parameters of the local area | | |
|---|---|---|
| $\sigma_{loc}$ | 2 | |
| $f_{el,sep,i} = \bar{f}_{el,sep}$ | 2.4 | |
| $f_{Q,sep,i} = \bar{f}_{Q,sep}$ | 1.22 | |
| Parameters of the chp plants | | |
| Technology | CC | BPST |
| $f_{F,chp,i}$ | 1.1 | 1.1 |
| $\sigma_{chp}$ | 1.2 | 0.2 |
| $\eta_{chp,i}$ | 78% | 85% |
| Parameters in the notation of the Appendix | | |
| $a$ | 0.4 | 0.9 |
| $b$ | 0.243 | 0.087 |
| $c$ | 2.361 | 0.393 |
| $d$ | 0.508 | 0.508 |
| $\eta^*$ | 0.590 | 0.776 |

with multiple energy-intensive products. For example, in addition to electric power we may have heat production at different temperature levels, compressed air, chilling, refrigeration, water desalination, and other energy-intensive products such as cement, steel, aluminum, and other materials or flows.

Let us denote by $E^{\text{facility k}}_{\text{product j}}$ the amount of product of j-th type delivered by the k-th production facility serving the area of interest. $f^{\text{facility k}}_{\text{product j}}$ denotes the corresponding primary energy factor. Products denoted by the same label j must be homogeneous in "energy quality", for example, heat delivered at different temperature levels should be identified by different labels. The different facilities may be separate production, cogeneration, or multi-generation. Each of them may consume a mix of different fuels or other primary energy resources, as well as a number of energy-intensive materials.

Let $F^{\text{facility k}}_{\text{primary}}$ denote the overall primary energy consumption of the k-th facility, given by

$$F^{\text{facility k}}_{\text{primary}} = \sum_i f^{\text{facility k}}_{\text{resource i}} E^{\text{facility k}}_{\text{resource i}} \quad (47)$$

where $E^{\text{facility k}}_{\text{resource i}}$ is the amount of resource (fuel or other, not necessarily expressed in unit of energy) of j-th type and $f^{\text{facility k}}_{\text{resource i}}$ the corresponding specific primary energy factor. For example, if resource k is iron ore and we express $E^{\text{facility k}}_{\text{resource i}}$ in ton of iron ore, then $f^{\text{facility k}}_{\text{resource i}}$ is in GWh of primary energy per ton of ore. Again, if resource k is waste heat recuperated from an industrial process and we choose to express $E^{\text{facility k}}_{\text{resource i}}$ as the exergy of the waste heat, then $f^{\text{facility k}}_{\text{resource i}}$ is in GWh of primary energy per GWh of recovered-waste-heat exergy.

Next we define the ratio of the amount of product j made in facility k to the overall primary energy consumed by facility k

$$\lambda^{\text{facility k}}_{\text{product j}} = \frac{E^{\text{facility k}}_{\text{product j}}}{F^{\text{facility k}}_{\text{primary}}} \quad (48)$$

and the local market share of facility k with respect to the production of j

$$\gamma^{\text{facility k}}_{\text{product j}} = \frac{E^{\text{facility k}}_{\text{product j}}}{\sum_n E^{\text{facility n}}_{\text{product j}}} \quad (49)$$

Finally, we denote by $f^{\text{loc. ave.}}_{\text{product j}}$ the average local primary energy factor for product j. In our STALPR method, allocation fractions are based on these average factors, therefore, the equivalent of Eqs. (23) is

$$\alpha^{\text{facility k}}_{\text{product j}} = \frac{f^{\text{loc. ave.}}_{\text{product j}} E^{\text{facility k}}_{\text{product j}}}{\sum_m f^{\text{loc. ave.}}_{\text{product m}} E^{\text{facility k}}_{\text{product m}}} = \frac{f^{\text{loc. ave.}}_{\text{product i}} \lambda^{\text{facility k}}_{\text{product j}}}{\sum_m f^{\text{loc. ave.}}_{\text{product m}} \lambda^{\text{facility k}}_{\text{product m}}} \quad (50)$$

the equivalent of Eqs. (24) are

$$f^{\text{facility k}}_{\text{product j}} = \frac{\alpha^{\text{facility k}}_{\text{product j}} F^{\text{facility k}}_{\text{primary}}}{E^{\text{facility k}}_{\text{product j}}} = \frac{\alpha^{\text{facility k}}_{\text{product j}}}{\lambda^{\text{facility k}}_{\text{product j}}} = \frac{f^{\text{loc. ave.}}_{\text{product j}}}{\sum_m f^{\text{loc. ave.}}_{\text{product m}} \lambda^{\text{facility k}}_{\text{product m}}} \quad (51)$$

and the average primary energy factors of the local area, i.e., the equivalent of Eq. (15) are given by

$$f^{\text{loc. ave.}}_{\text{product j}} = \frac{\sum_k f^{\text{facility k}}_{\text{product j}} E^{\text{facility k}}_{\text{product j}}}{\sum_n E^{\text{facility n}}_{\text{product j}}} = \sum_k f^{\text{facility k}}_{\text{product j}} \gamma^{\text{facility k}}_{\text{product j}} \quad (52)$$

Finally, substituting Eq. (51) into (52), we obtain the system of equations

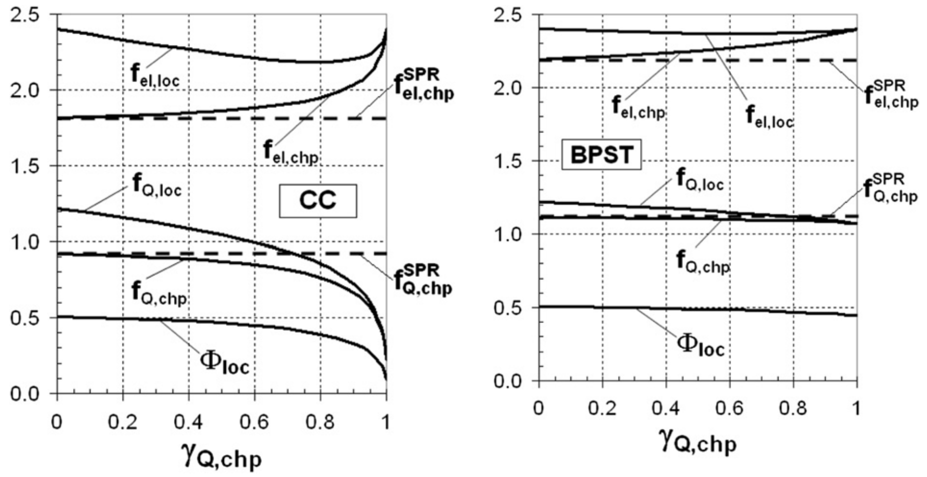


**Fig. 3.** $\Phi_{loc}$ and primary energy factors plotted as functions of the parameter $\gamma_{Q,chp}$ for a local area with $\sigma_{loc} = 2$ and the same values of all the other parameters as assumed in Fig. 2.

$$1 = \sum_k \frac{\gamma^{\text{facility k}}_{\text{product j}}}{\sum_m f^{\text{loc. ave.}}_{\text{product m}} \lambda^{\text{facility k}}_{\text{product m}}} \tag{53}$$

which for given values of the $\lambda^{\text{facility k}}_{\text{product j}}$ and the $\gamma^{\text{facility k}}_{\text{product j}}$ determines the values of the $f^{\text{loc. ave.}}_{\text{product m}}$'s which in turn can be used in the previous equations to find all other factors and fractions.

In practice, we conclude and suggest that a self-consistent cogeneration regulation based on the proposed STALPR fair allocation method is easily obtained by simply adopting Eqs. (47)–(49), (53).

As a validation example, we may use the general notation introduced in this section to reformulate the case considered in Section 3, where we have two products (1 = el; 2 = Q) and three facilities (1 = el,sep; 2 = Q,sep; 3 = chp) of which the first two are single production, therefore $\lambda_1^1 = 1/f_{\text{el,sep}}$, $\lambda_2^2 = 1/f_{\text{Q,sep}}$, $\lambda_2^1 = 0$, $\gamma_2^1 = 0$, $\lambda_1^2 = 0$, and $\gamma_1^2 = 0$, so that Eqs. (53) become

$$1 = \frac{\gamma_1^1}{f_1^{\text{loc}}\lambda_1^1} + \frac{\gamma_1^3}{f_1^{\text{loc}}\lambda_1^3 + f_2^{\text{loc}}\lambda_2^3} \tag{54}$$

$$1 = \frac{\gamma_2^2}{f_2^{\text{loc}}\lambda_2^2} + \frac{\gamma_2^3}{f_1^{\text{loc}}\lambda_1^3 + f_2^{\text{loc}}\lambda_2^3} \tag{55}$$

These equations can be easily translated in the notation of Section 3 by recalling that $\gamma_1^1 = \gamma_{\text{el,sep}}$, $\gamma_1^3 = 1-\gamma_{\text{el,sep}}$, $\gamma_2^2 = \gamma_{\text{Q,sep}}$, $\gamma_2^3 = 1-\gamma_{\text{Q,sep}}$ and noticing that $\lambda_1^1 = 1/f_{\text{el,sep}}$, $\lambda_2^2 = 1/f_{\text{Q,sep}}$, $\lambda_2^3 = \eta_{\text{chp}}/f_{\text{F,chp}}(\sigma_{\text{chp}}+1)$, $\lambda_1^3 = \sigma_{\text{chp}}\lambda_2^3$ to yield

$$1 = \frac{f_{\text{el,sep}}\gamma_{\text{el,sep}}}{f_{\text{el,loc}}} + \frac{f_{\text{F,chp}}\left(\sigma_{\text{chp}}+1\right)\left(1-\gamma_{\text{el,sep}}\right)}{\eta_{\text{chp}}\left(f_{\text{el,loc}}\sigma_{\text{chp}} + f_{\text{Q,loc}}\right)} \tag{56}$$

$$1 = \frac{f_{\text{Q,sep}}\gamma_{\text{Q,sep}}}{f_{\text{Q,loc}}} + \frac{f_{\text{F,chp}}\left(\sigma_{\text{chp}}+1\right)\left(1-\gamma_{\text{Q,sep}}\right)}{\eta_{\text{chp}}\left(f_{\text{el,loc}}\sigma_{\text{chp}} + f_{\text{Q,loc}}\right)} \tag{57}$$

It is now easy to verify that equating the right hand sides of Eqs. (56) and (57) yields, after few rearrangements, Eq. (30) for $\Phi_{\text{loc}} = f_{\text{Q,loc}}/f_{\text{el,loc}}$

## 6. Conclusions

Cogeneration technologies, i.e., the combined productions in a single facility of a mix of two or more different energy-intensive goods, are capturing higher and higher fractions of the energy market because they produce important savings in primary energy and avoided emissions. Cogeneration regulations are being developed in order to allocate such benefits in a fair way between the different cogenerated goods. Current regulations are based on the Separate Production Reference (SPR) method whereby the primary energy consumption and emissions in a cogeneration facility are allocated based on the relative consumptions and emissions that are required to produce the same energy-intensive goods in a prescribed reference set of separate-production facilities.

In this paper, we have shown that the SPR method provides unfair, distorted figures arising from an intrinsic inconsistency which becomes increasingly important as cogeneration gains higher fractions of the energy market in a given local area. Our observation stems from the fact that cogeneration facilities are almost always part of a local production scenario, i.e., a local area (district, city, regional, national, interstate) energy system providing end users with electricity, residential heating or air-conditioning, industrial process steam, desalinated water, and/or other energy-intensive products. The overall savings obtained by introducing a given combined production plant in a certain local area, have a relative impact which depends on the pre-existing local situation, therefore it is unfair to allocate the savings among the cogenerated goods without taking into proper account the local area situation. It is conceivable that a given combined production facility could constitute an improvement for one local area but an aggravation for another local area. The inconsistency of the SPR method stems from the fact that it assigns the same primary energy allocation to the facility, regardless of the parameters of the local area in which it operates.

To resolve the issue, we propose a natural extension of the SPR method so as to take in due and fair account the local scenario in which a given cogeneration facility operates. The result, that we call the Self-Tuned Average-Local-Productions Reference (STALPR) method, is self-consistent in that the allocation parameters are self-tuned in terms of the energy scenario of the given local area of interest. This is achieved by an adaptive and self-tuned allocation whereby the fuel savings obtained by cogeneration are shared among the cogenerated products on the basis of the average primary energy factors of such products in the given local area, including the cogeneration facility of interest.

The self-consistency of the STALPR method is gained at the expense of a slightly higher complexity of the formulation with respect to the standard SPR method. However, we show by examples that the numerics are simple and readily tractable even in the general case of many goods and many cogeneration facilities in the given area. The differences between STALPR and SPR allocations are important in local areas with relatively high levels of cogeneration.

Importantly, the STALPR method puts forward and embodies the novel idea that if the benefits of cogeneration (in this paper we focused on fuel savings) are to be allocated among cogenerated products in a permanently fair way, the allocation must be adaptive and dynamically tied to the evolving local area production scenario through the (evolving) average primary energy factors of the cogenerated products. As a result, each existing cogeneration facility in the local area must revise its allocation parameters whenever a new cogeneration or separate production facility is installed in the local area and/or an old one is removed.

From a fundamental point of view, we note that since the historical developments of energy technologies evolve along learning curves [31] that are headed towards approaching the highest possible production and conversion efficiencies compatible with thermodynamic limitations, it is reasonable to foresee that the dynamic allocation fractions generated by the STALPR method will correspondingly converge towards the static allocation fractions defined by the exergy-based allocation method [1–3,9,17].

In a forthcoming paper, we will show how the principle of self-tuning via average-local area parameters introduced here to fairly allocate fuel savings obtained via cogeneration, can be readily implemented to achieve a similar self-tuned, adaptive, fair allocation methodology also for the carbon dioxide emission savings that obtain from cogeneration. Discussion of such methodology is particularly timely because the EU Commission is currently revising the regulatory principles guiding the development of the EU emission allowance trading scheme [28,29].

## Acknowledgments

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## Appendix

Recalling Eq. (26),

$$\gamma_{el,chp} = \frac{\sigma_{chp}\gamma_{Q,chp}}{\sigma_{loc}}$$

we can rewrite Eq. (30) in compact form as

$$f(y,x,a,b,c) = [1-(1-a)x]y^2 + [(c+1)abx - 1 + (1-a)x + (1-x)c]y - (1-x)c = 0$$

where we define the following new variables

$$a = 1 - \frac{\sigma_{chp}}{\sigma_{loc}}$$
$$b = 1 - \frac{\eta^*}{\eta_{chp}}$$
$$c = \frac{\sigma_{chp}}{d}$$
$$d = \frac{\bar{f}_{Q,sep}}{\bar{f}_{el,sep}}$$
$$\eta^* = \frac{\left(\sigma_{chp}+1\right)f_{F,chp}}{\bar{f}_{el,sep}\sigma_{chp} + \bar{f}_{Q,sep}} = \frac{f_{F,chp}}{\bar{f}_{el,sep}}\frac{cd+1}{(c+1)d}$$
$$x = \gamma_{Q,chp}$$
$$y = \frac{\Phi_{loc}}{d}$$

and we note that their meaningful ranges of values are

$a \le 1$
$b \le 1$
$c > 0$
$d > 0$
$0 \le 1 - x = \gamma_{Q,sep} \le 1$
$0 \le 1 - (1-a)x = \gamma_{el,sep} \le 1$
$0 \le x \le \min\left(1, \frac{1}{1-a}\right)$
$x_{max} = 1$ for $a < 0$ and $x_{max} = 1/(1-a)$ for $a > 0$
$y > 0$
$\eta^* > 0$

Within these ranges of values, except at $x = 1$ and $x = 1/(1-a)$, we have $[1-(1-a)x](1-x)c > 0$, therefore, the second order equation in $y$ defined by $f(y,x,a,b,c) = 0$ has only one positive root. Moreover, for the same reason, $\partial f/\partial y > 0$[2] (for $x = 0$ and $y = 1$, $\partial f/\partial y = c+1$).

Eqs. (25) rewrite as

$$f_{Q,chp} = \frac{(c+1)y}{c+y}\frac{\eta^*}{\eta_{chp}}\bar{f}_{Q,sep} \text{ and } f_{el,chp} = \frac{c+1}{c+y}\frac{\eta^*}{\eta_{chp}}\bar{f}_{el,sep} \quad \text{(A1)}$$

In particular, for $y = 1$ we have

$$f_{Q,chp} = \frac{\eta^*}{\eta_{chp}}\bar{f}_{Q,sep} \text{ and } f_{el,chp} = \frac{\eta^*}{\eta_{chp}}\bar{f}_{el,sep} \quad \text{(A2)}$$

Notice that $y = 1$ implies $(c+1)abx = 0$. Viceversa, $x = 0$ implies $y = 1$ ($\Phi_{loc} = \bar{f}_{Q,sep}/\bar{f}_{el,sep}$). Indeed $x = 0$ implies $(y-1)(y+c) = 0$, but $y = -c$ is out of meaningful range. Recalling the definition of $\eta^*$ and using Eqs. (19) and (20), after a few trivial rearrangements, Eqs. (A2) reduce to the same formulation of the primary energy factors of the SPR approach defined by Eqs. (10), meaning that when no cogeneration plants are present in the local area the STALPR model is equivalent to SPR.

As noted in Section 3, if $\Phi_{loc}$ increases then $f_{Q,chp}$ increases and $f_{el,chp}$ decreases. It is therefore interesting to investigate further the function $y = y(x,a,b,c)$, defined by Equation $f(y,x,a,b,c) = 0$. Since the differential of $f(y,x,a,b,c)$ is identically zero, we may obtain the partial derivatives of $y(x,a,b,c)$. In particular,

$$\frac{\partial y}{\partial x} = -\frac{\partial f/\partial x}{\partial f/\partial y} = \frac{(1-a)y^2 + [c-(1-a)-(c+1)ab]y - c}{\partial f/\partial y}$$

For $x = 0$ (no cogeneration plants in the local area and, therefore, as seen above, $y = 1$), this derivative is $-ab$ and it is zero when either $\sigma_{chp} = \sigma_{loc}$ or $\eta_{chp} = \eta^*$. In both cases, $y = 1$ holds for all values of $x$, i.e., $\Phi_{loc} = \bar{f}_{Q,sep}/\bar{f}_{el,sep}$ holds for all values of $\gamma_{Q,chp}$. However, the latter case ($\eta_{chp} = \eta^*$) yields, by Eqs. (A2), $f_{Q,chp} = \bar{f}_{Q,sep}$ and $f_{el,chp} = \bar{f}_{el,sep}$, therefore, it represents the case of a (poor) cogeneration plant that consumes the same primary energy as the separate production facilities. Indeed, for a chp plant to be of practical interest, i.e., beneficial to the local area, it must have $\eta_{chp} > \eta^*$ or equivalently $b > 0$. This can also be seen more directly as follows. If the heat and electricity produced by the chp plant replace the same amounts, respectively, of separately produced heat and electricity, the condition for the replacement to be beneficial is that it must reduce the overall primary energy consumption of the local area, that is,

$$\left(f_{el,chp} - \bar{f}_{el,sep}\right)E_{el,chp} + \left(f_{Q,chp} - \bar{f}_{Q,sep}\right)E_{Q,chp} < 0 \quad \text{(A3)}$$

This can be rewritten as

$$(c+1)(c+y)bd^2x > 0$$

which in all cases imposes $b > 0$.

When $ab \ne 0$, at $y = 1$ and $x = 0$ the derivative is positive (i.e., $\Phi_{loc}$ increases with $\gamma_{Q,chp}$) if $ab < 0$, i.e., if $a < 0$ ($\sigma_{chp} > \sigma_{loc}$) since in all cases of practical interest $b > 0$. In this case, $y$ is increasing with $x$ until, for the maximum meaningful value $x_{max} = 1/(1-a)$, it reaches the maximum value $y = c/[c-(c+1)b]$ if $(c+1)b < c$ or $y = \infty$ if $(c+1)b \ge c$, independent of the value of $a$ (as long as $a < 0$). Recalling the definitions of $y$, $b$, and $c$, and using Eqs. (25), such maximum values correspond to the following conclusions that apply when $\sigma_{chp} > \sigma_{loc}$ and $\gamma_{el,chp} = 1$:

If $\bar{f}_{Q,sep}E_{Q,chp} < f_{F,chp}E_{F,chp}$ $\left(\text{i.e.,} \bar{f}_{Q,sep} < \frac{f_{F,chp}\left(\sigma_{chp}+1\right)}{\eta_{chp}}\right)$ then

$$\Phi_{loc} = \frac{\bar{f}_{Q,sep}\eta_{chp}\sigma_{chp}}{f_{F,chp}\left(\sigma_{chp}+1\right) - \bar{f}_{Q,sep}\eta_{chp}}$$
$$f_{Q,chp} = \bar{f}_{Q,sep}$$
$$f_{el,chp} = \frac{f_{F,chp}\left(\sigma_{chp}+1\right) - \bar{f}_{Q,sep}\eta_{chp}}{\eta_{chp}\sigma_{chp}} \quad \text{(A4)}$$

[2] To see this, we write the second order equation as $f(y) = Ay^2 + By - C = 0$ where $A \ge 0$ and $C \ge 0$. Then, since $\sqrt{B^2+4AC} > B$, $y = (-B + \sqrt{B^2+4AC})/2A$ is the only positive root. Therefore, $\partial f/\partial y = 2Ay + B = \sqrt{B^2+4AC} > 0$.

$$\text{If } \bar{f}_{Q,sep}E_{Q,chp} > f_{F,chp}E_{F,chp}\left(\text{i.e., } \bar{f}_{Q,sep} > \frac{f_{F,chp}\left(\sigma_{chp}+1\right)}{\eta_{chp}}\right) \text{ then}$$

$$\begin{aligned}
\Phi_{loc} &= \infty \\
f_{Q,chp} &= \frac{f_{F,chp}\left(\sigma_{chp}+1\right)}{\eta_{chp}} \\
f_{el,chp} &= 0
\end{aligned} \qquad (A4')$$

Notably the condition $(c+1)b \geq c$, for which Eqs. (A4′) hold true, corresponds to the extreme situation in which: (1) the entire electricity consumption in the local area is cogenerated, and (2) the non-cogenerated heat is produced by relatively low-efficiency heat facilities, consuming more primary energy than the cogeneration facilities in the local area would consume to cogenerate the same amount of heat.

For $a > 0$, $y$ is decreasing with $x$, until it reaches the minimum value $y = 1-(c+1)b$ if $(c+1)b < 1$ or $y = 0$ if $(c+1)b \geq 1$, in either case independent of the value of $a$ (as long as $a > 0$). Again by recalling the definitions of $y$, $b$, $c$ and combining them with Eqs. (25) such minimum values correspond to the following conclusions that apply when $\sigma_{chp} < \sigma_{loc}$ and $\gamma_{Q,chp} = 1$:

$$\text{If } \bar{f}_{el,sep}E_{el,chp} < f_{F,chp}E_{F,chp}\left(\text{i.e., } \bar{f}_{el,sep} < \frac{f_{F,chp}\left(\sigma_{chp}+1\right)}{\eta_{chp}\sigma_{chp}}\right) \text{ then}$$

$$\begin{aligned}
\Phi_{loc} &= \frac{f_{F,chp}\left(\sigma_{chp}+1\right) - \bar{f}_{el,sep}\eta_{chp}\sigma_{chp}}{\bar{f}_{el,sep}\eta_{chp}} \\
f_{Q,chp} &= \frac{f_{F,chp}\left(\sigma_{chp}+1\right) - \bar{f}_{el,sep}\eta_{chp}\sigma_{chp}}{\eta_{chp}} \\
f_{el,chp} &= \bar{f}_{el,sep}
\end{aligned} \qquad (A5)$$

$$\text{If } \bar{f}_{el,sep}E_{el,chp} > f_{F,chp}E_{F,chp}\left(\text{i.e., } \bar{f}_{el,sep} > \frac{f_{F,chp}\left(\sigma_{chp}+1\right)}{\eta_{chp}\sigma_{chp}}\right) \text{ then}$$

$$\begin{aligned}
\Phi_{loc} &= 0 \\
f_{Q,chp} &= 0 \\
f_{el,chp} &= \frac{f_{F,chp}(\sigma_{chp}+1)}{\eta_{chp}\sigma_{chp}}
\end{aligned} \qquad (A5')$$

The condition $(c+1)b \geq 1$, for which Eqs. (A5′) hold true, corresponds to the extreme situation in which: (1) the entire heat consumption in the local area is cogenerated, and (2) the non-cogenerated electricity is produced by relatively low-efficiency power plants, consuming more primary energy than the cogeneration facilities in the local area would consume to cogenerate the same amount of electricity.

Examples of the two particular cases relevant to Eqs. (A5′) and (A4′) are provided in Fig. A1.

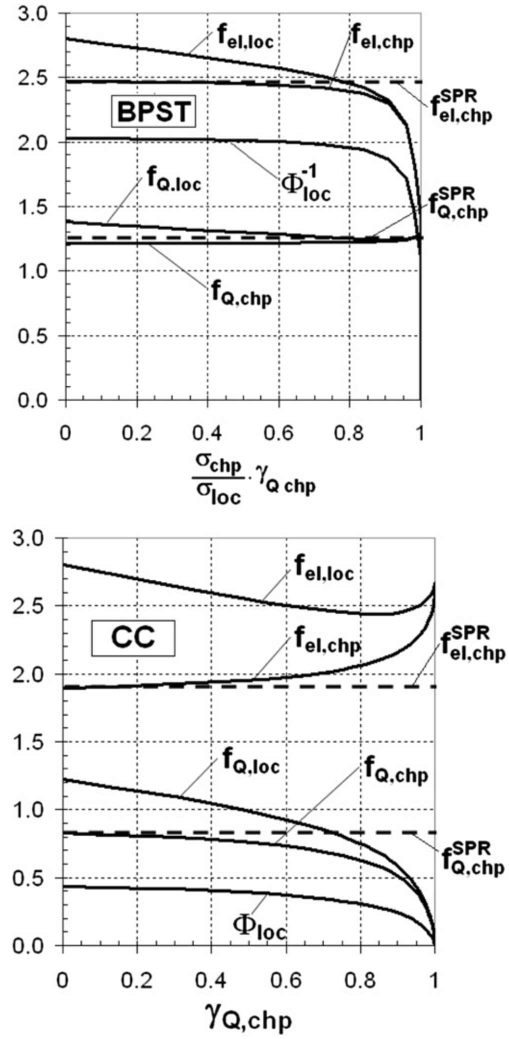


**Figure A1.** $\Phi_{loc}$ and primary energy factors plotted as functions of the parameter $\gamma_{Q,chp}$ for two particular cases corresponding to Eqs. (A4′) (top) and Eqs. (A5′) (bottom), respectively. Top: local area with $\sigma_{loc} = 0.06$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.38$ and CHP facilities based on steam cycle technology with back-pressure steam turbine (BPST) with $f_{F,chp} = 1.1$, $\sigma_{chp} = 0.065$, and $\eta_{chp} = 0.85$. Bottom: local area with $\sigma_{loc} = 2$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.22$, and CHP facilities based on combined cycle (CC) technology with $f_{F,chp} = 1.1$, $\sigma_{chp} = 1.2$, and $\eta_{chp} = 0.78$. Dashed lines in both figures refer to the primary energy factors calculated with the classical SPR method. Note that for the BPST case, here we plot $\Phi^{-1}_{loc}$ instead of $\Phi_{loc}$ since $\Phi_{loc} \to \infty$ as $\gamma_{Q,chp}$ tends to its maximum value $\sigma_{loc}/\sigma_{chp} = 0.923$.

Of course, we expect that as $x$ begins to depart from zero, not only $y$ departs from zero, but the curve will bend. Indeed, the second partial derivative of $y(x,a,b,c)$ is

$$\frac{\partial^2 y}{\partial x^2} = -\frac{\partial^2 f/\partial x^2}{\partial f/\partial y} + \frac{\partial f/\partial x}{(\partial f/\partial y)^2}\partial^2 f/\partial x \partial y =$$

$$\frac{(1-a)y^2 + [c-(1-a)-(c+1)ab]y - c}{(\partial f/\partial y)^2}[2(1-a)y-(1-a)+c-(c+1)ab]$$

and, at $y = 1$ and $x = 0$, it is equal to $-ab + a^2b^2 + a^2b/(c+1)$, hence, for $a < 0$ it is positive so the curve at $y = 1$ and $x = 0$, which is increasing, bends upwards whereas, for $0 < a < (c+1)/(b+bc+1)$ it is negative so the curve at $y = 1$ and $x = 0$, which is decreasing, bends downwards.

Finally, for given $x > 0$ we may ask if $y$ is an increasing or decreasing function of the other parameters. For example, as a function of the efficiency of the chp plant,

$$\frac{\partial y}{\partial b} = -\frac{\partial f/\partial b}{\partial f/\partial y} = -\frac{(c+1)axy}{\partial y/\partial y}$$

from which we see that $(\partial f/\partial b)a < 0$ throughout, i.e., for $a < 0$ it increases with $b$ while for $a > 0$ it decreases with $b$ as evidenced by the curves shown in Figures A2 and A3 respectively.

Again, as a function of $\sigma_{loc}$ (which determines the value of the parameter $a$ in case of chp plants with fixed electric index $\sigma_{chp}$)

$$\frac{\partial y}{\partial a} = -\frac{\partial f/\partial a}{\partial f/\partial y} = \frac{[1-(c+1)b-y]xy}{\partial f/\partial y}$$

We have seen above that for $a > 0$, $y > 1-(c+1)b$ for every $x$, therefore, $(\partial f/\partial a) < 0$. For $a < 0$, $y$ starts at $y = 1$ for $x = 0$ and is increasing with $x$ therefore, again, $y \geq 1 > 1-(c+1)b$ and so we see that $(\partial f/\partial a) < 0$ throughout, which clearly implies that $y$ decreases with $a$, as shown by the curves reported in Figure A4.

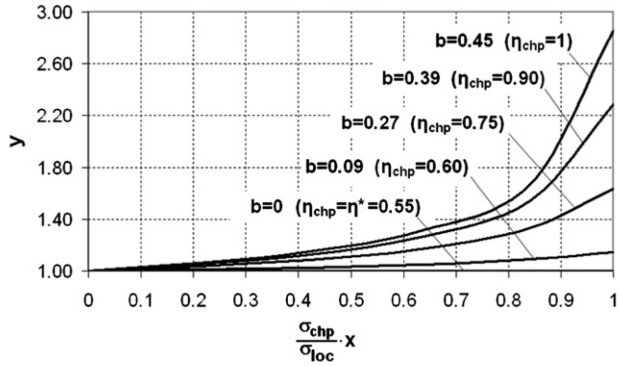


**Figure A2.** Plots of $y$ versus $(\sigma_{chp}/\sigma_{loc})x$ for prescribed values of $a < 0$, $c$, $d$, and different values of $b$. Here we assumed the following fixed values: $\sigma_{chp} = 1$, $\sigma_{loc} = 0.475$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.22$, $\bar{f}_{F,chp} = 1.1$. Therefore, $\eta^* = 0.5473$, $a = -1.1053$, $b = 1-\eta^*/\eta_{chp}$, $c = 2.295$, $d = 1.22/2.8 = 0.4357$.

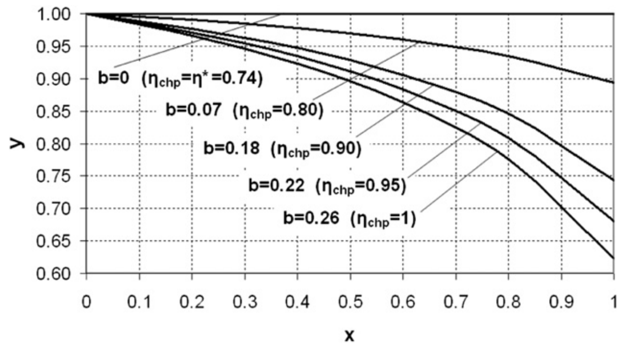


**Figure A3.** Plots of $y$ versus $x$ for prescribed values of $a > 0$, $c$, $d$, and different values of $b$. Here the assumed fixed values are: $\sigma_{chp} = 0.2$, $\sigma_{loc} = 0.475$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.22$, $\bar{f}_{F,chp} = 1.1$. Therefore, $\eta^* = 0.7416$, $a = 0.5789$, $b = 1-\eta^*/\eta_{chp}$, $c = 0.4590$, $d = 0.4357$.

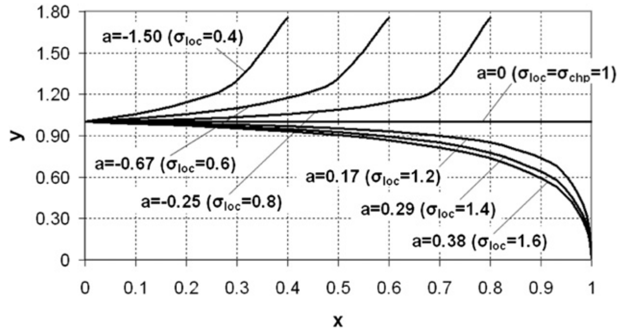


**Figure A4.** Plots of $y$ versus $x$ for prescribed values of $b$, $c$, $d$, and different values of $a$. Here the assumed parameters are: $\sigma_{chp} = 1$, $\eta_{chp} = 0.78$, $\bar{f}_{el,sep} = 2.8$, $\bar{f}_{Q,sep} = 1.22$, $\bar{f}_{F,chp} = 1.1$. Therefore, $\eta^* = 0.5473$, $a = 1-1/\sigma_{loc}$, $b = -0.4253$, $c = 2.295$, $d = 0.4357$.

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