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Allocating electricity production from a hybrid fossil-renewable power plant among its multi primary resources

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ABSTRACT

The interest in hybrid power production facilities, based on the integration of renewable resources and conventional fossil fuels, is rapidly rising. The question of what fraction of the electricity produced in such facilities is to be considered as produced from the renewable resources is still being debated. We show that the conventional Fossil-Centered-Solar-Share method and the Exergy-based method lead to unfair allocations that may result in unfair access to subsidies granted to renewable electricity. We propose a more balanced Single-Resource-Separate-Production-Reference (SRSPR) allocation method based on prescribed reference partial primary energy factors chosen by some authority to represent reference efficiencies of non-hybrid power production from the same renewable and fossil resources used by the hybrid facility. We then show that as hybridization gains higher fractions of the local energy market, the SRSPR method may still result in somewhat unfair allocations leading to local market distortions. To overcome this drawback, we formulate a more consistent Self-Tuned-Average-Local-Productions-Reference (STALPR) allocation method whereby the electricity allocation fractions are based on the average partial primary energy factors of the actual energy portfolio of the local area that includes the hybrid plant itself. Results are illustrated with reference to a solar-integrated combined cycle facility.

1. Introduction

In many industrial and manufacturing facilities, higher efficiencies are achieved by integrating the production of a mix of different goods and/or using a mix of different resources and/or raw materials. Such facilities are therefore either multi-generation (cogeneration) or multi-resource, or both [1]. Whenever the process consumes fossil primary energy, the achieved higher efficiencies also imply a reduction of the associated CO_2 emissions [2]. Thus many of these solutions will play a prominent role in future production scenarios. For instance, one can use a single resource, such as coal or natural gas, to produce other fuels, such as liquid or gaseous fuels, chemicals such as methanol, and electricity. One can also use multiple sources, such as coal and biomass, solar energy and fuels, to produce electricity. In these cases, a question that often arises is how to allocate fractions of the single input resource to the different outputs, or how to allocate fractions of the single output to the different input sources. This is important when incentives are used to promote the use of particular resources, or to promote efficiency enhancement in the use of a single resource. In Ref. [1], we focused on a single-resource multi-generation facility, such as a gas-fired heat-and-power cogenerator, and addressed the problem of defining a ‘fair’ method to allocate fractions of the single resource among the different products, that is how much of the gas consumption should be allocated to the production of heat and to the production of electricity. In other words, we showed how to determine the primary energy factors of heat and electricity produced in a cogeneration facility.

In this paper, we focus on the conjugate problem, that is, the single-product multiple-resource facilities, and address the problem of defining a ‘fair’ method to allocate fractions of the product to the different resources it consumes. An example of particular interest in the energy sector, which motivates the present work and is taken here as our case study, is that of the so-called ‘hybrid’ power production facilities. In these power plants, one combines and integrates renewable energy resources, like solar energy, and fossil fuel in conventional energy conversion systems such as steam

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cycle, gas turbine cycle, or combined cycle power plants. These hybrid energy systems are receiving much attention because they provide an economic, sustainable, and reliable use of renewable energy under all load-demand conditions compared to renewable resource-only facilities. The need to reduce CO_2 emissions that contribute to global warming, and the accelerated consumption and the rise in the prices of fossil fuels have motivated many countries to adopt energy policies that promote the use of renewable energy and hybrid power plants [3]. In some cases, there are mandates for producing a fraction of the electricity from renewable sources. Since such policies usually provide subsidies to "renewable electricity", an important question arises as to how in a hybrid facility one should compute the fraction of the output electricity that qualifies for the subsidies reserved to the use of renewable resources. In other words, we need a 'fair' method to allocate the electricity produced by the facility among the renewable energy and fossil fuel it consumes in the process.

In the present analysis, we focus on the electricity allocation in hybrid fossil-solar facilities, as they represent a viable solution for dependable use of solar energy, while overcoming the hourly, diurnal and seasonal variability without the need for storage. Besides being a good answer to the intermittency problem, the integration of the two sources can be done in such a way as to maximize the efficiency of the overall plant. The methodology we describe here can be used for other configurations, such as geothermal-fossil, geothermal-solar-fossil, biomass-fossil, etc. In all cases, the overall objective is to estimate the fractional contribution of the renewable source to the electricity produced by the plant in a way compatible with the local electricity production portfolio. We will show that resolving this allocation problem solves also the problem of determining the fossil-fuel-to-electricity conversion efficiency and the solar-to-electricity conversion efficiency of the hybrid power plant. Such conversion efficiencies can help the regulator to provide fair incentives to promote the use of hybrid technologies instead of single-resource technologies, much like incentives are used to promote the use of cogeneration instead of separate-production heat and power plants.

We introduce two methods of allocation. The first, that we call Single-Resource-Separate-Production-Reference (SRSPR) allocation method, requires the local energy authority or regulator to set a reference efficiency for each resource-product pair and periodically update them. For example, for the case of a hybrid facility using natural gas and solar radiation, the SRSPR method requires a reference efficiency for solar-to-electricity conversion and a reference efficiency for gas-to-electricity conversion. This method is the conjugate of the classical method of allocation used in cogeneration that, in Ref. [1], we called Separate-Production-Reference method (SPR). The second method we propose is an adaptive marginal allocation method (that we call STALPR). It is slightly more elaborate but requires no prescribed reference efficiencies. Rather, it is self-tuned and context-dependent by replacing the reference efficiencies with the actual average efficiencies of the power plant portfolio in the local area, including the hybrid facility itself. By emphasizing and self-adapting to the local scenario, this method addresses the role of local differences which are particularly important in the case of solar-fossil hybrid facilities since the performance of a renewable technology is strongly impacted by the local geographical and weather conditions. The STALPR method applies irrespective of the hybrid power plant technology and only considers the overall performance parameters of the existing plants in the area, namely, the existing renewable-only and fossil-fuel-only facilities, and the hybrid facilities, including the one under consideration.

Different hybrid fossil-solar configurations are possible and many have been the subject of several studies in recent years, dealing with technological integration issues as well as with the metrics necessary to evaluate the hybrid plant from both the thermodynamic and the economic points of view [4]. Three main schemes are currently considered for the hybridization: the solarized gas turbines, the hybrid combined cycles, and the solar reforming systems. Solarized gas turbines use a concentrating solar power system to preheat the compressed air (up to temperature around 1070 K) before it enters the combustion chamber. Several configurations of solar hybrid gas turbine cycles in the low to medium power range are examined in terms of performance and costs in Refs. [5,6]. Solar energy is typically incorporated into a combined cycle in two ways: in the gas turbine as in solarized gas turbines or as supplemental solar heat to the bottoming Rankine cycle. A comprehensive performance and economic analysis of various solutions adopted for hybrid combined cycles can be found in Refs. [7–9]. Finally with the solar reforming systems a solar reformer is used to reform a traditional fossil fuel (like methane) and then the reformate from the solar reformer is used as the fuel in a traditional fossil fuel power cycle. References [10,11] deal with such application in the cases of a gas turbine cycle and a combined cycle respectively.

Moreover, different metrics have been used to determine the relative performance of hybridized plants, using modified forms of the first law and second law efficiencies, or focused on the contribution of the solar energy such as the incremental solar efficiency based on the input or output energy. We will show that the latter metric, which implies the allocation rule used in the Fossil-Centered-Solar-Share (FCSS) method, provides an unfair apportionment of the benefits of hybridization among the two primary energy resources used in the facility.

For the purposes of testing the different allocation methods described in the present paper, we adopt the technology of Solar-Integrated Combined-Cycle System (SICCS) with parabolic trough solar field as our case study. Presently, this represents a mature hybridization solution for large-scale application [12] as confirmed by the recent implementation in few pilot plants such as the Abengoa plant built in Algeria [13] and the Martin Next Generation Solar Energy Center, completed in 2010 in Florida [14].

We use this case study to demonstrate the value of the proposed approach.

The allocation problem we consider here is conjugate to the one we described in Ref. [1], and our presentation will emphasize the analogies wherever possible. In Section 2 we formally define the general allocation problem and the 'conventional' FCSS method so far adopted or under consideration by the regulators. We also introduce two methods that to our knowledge have not yet been considered in this context, namely, the Exergy-based Reversible-Reference (ExRR) method and the Single-Resource-Separate-Production-Reference (SRSPR) allocation method. We conclude Section 2 with a critique of these three 'classical' methods. In Section 3 we propose our adaptive Self-Tuned-Average-Local-Productions-Reference method (STALPR). In Section 4 we define a realistic case study that we then use to compare the results of the SRSPR and the STALPR methods. In Section 5 we define the primary energy savings parameters characterizing a hybrid power plant and discuss the rationale of possible incentive policies for hybrid plants. Section 6 gives the conclusions. The Appendix details the dual relationship between the allocation problem discussed in the present paper and the conjugate one addressed in Ref. [1].

2. Allocation problem definition for a hybrid power plant

We consider the allocation of a single-product (electricity) among the different input resources (solar energy and natural gas) of the hybrid power plant sketched in Fig. 1. On a yearly basis the hybrid

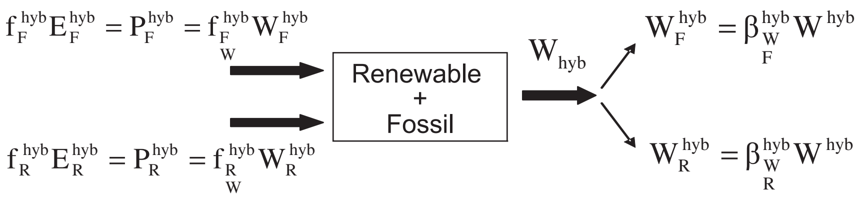

Fig. 1. Allocation problem definition for a hybrid power plant.

plant consumes a quantity P_R^{hyb} of renewable primary[1] energy and P_F^{hyb} of fossil primary energy, and delivers a total of W_{hyb} of electrical energy. Notice that we use the symbol P for primary energy associated with each input of a power plant and the symbol E for the corresponding energy. The subscript and superscript 'hyb' refers to the hybrid facility, the subscript 'R' stands for 'renewable source', 'F' for 'fossil resource'.

Since the incentive policies usually provide subsidies to the electricity produced by renewable sources only, a fair criterion is necessary to identify the renewable share of the produced electricity, i.e., we need to determine how to split the total electrical energy W_{hyb} into two parts: W_R^{hyb} and W_F^{hyb} representing the renewable and fossil shares, respectively. For this purpose we define the electricity allocation fractions β's representing the two unknowns of the resource allocation problem

$$\beta^{hyb}_{\substack{W\\R}} = \frac{W_R^{hyb}}{W_{hyb}} \text{ and } \beta^{hyb}_{\substack{W\\F}} = \frac{W_F^{hyb}}{W_{hyb}} \tag{1}$$

where of course

$$\beta^{hyb}_{\substack{W\\R}} + \beta^{hyb}_{\substack{W\\F}} = 1 \tag{2}$$

The subscript 'W' here stands for 'work'. For the purposes of the present paper, we could dispose of this subscript and simplify slightly the notation, however we prefer to keep the current notation for future reference and compatibility with notation we shall use in a forthcoming paper in which we address the allocation problem for a hybrid two resources (R and F) two products (W and Q) cogeneration facility, where 'Q' stands for 'heat'. The electricity allocation fraction $\beta^{hyb}_{\substack{W\\R}}$ represents the fraction of the overall electricity production in the hybrid plant that is to be considered as obtained from the renewable resource used by the hybrid facility.

Next, we define the primary energy factors[2] of the resources used by the hybrid facility, which account for the primary energy consumed in making a unit of that kind of energy available at the plant:

$$f_R^{hyb} = \frac{P_R^{hyb}}{E_R^{hyb}} \text{ and } f_F^{hyb} = \frac{P_F^{hyb}}{E_F^{hyb}} \tag{3}$$

where E_F^{hyb} is the energy (based on lower heating value) of the fossil fuel and E_R^{hyb} is the renewable energy input to the hybrid facility.[3] Moreover, we define the 'partial primary energy factors' $f^{hyb}_{\substack{R\\W}}$ and $f^{hyb}_{\substack{F\\W}}$, which account for the portions of the overall electricity produced that we allocate to either the renewable or the fossil resource

$$f^{hyb}_{\substack{R\\W}} = \frac{P_R^{hyb}}{W_R^{hyb}} \text{ and } f^{hyb}_{\substack{F\\W}} = \frac{P_F^{hyb}}{W_F^{hyb}} \tag{4}$$

The system of seven Eqs. (1)–(4) can be solved for the eight unknowns $\beta^{hyb}_{\substack{W\\R}}$, $\beta^{hyb}_{\substack{W\\F}}$, P_R^{hyb}, P_F^{hyb}, $f^{hyb}_{\substack{R\\W}}$, $f^{hyb}_{\substack{F\\W}}$, W_R^{hyb}, and W_F^{hyb} in terms of given values for W_{hyb}, E_R^{hyb}, E_F^{hyb}, f_R^{hyb}, f_F^{hyb} provided we supply an additional reasonable relation between $f^{hyb}_{\substack{R\\W}}$ and $f^{hyb}_{\substack{F\\W}}$. Such relation furnishes the closure of the problem and characterizes the allocation method.

To be more explicit, it is convenient to recast the above equations in terms of the following new variables

$$\chi_{res}^{hyb} = \frac{f_F^{hyb}}{f_R^{hyb}} \tag{5}$$

$$\sigma_{hyb} = \frac{P_F^{hyb}}{P_R^{hyb}} \tag{6}$$

$$\eta_{hyb} = \frac{W_{hyb}}{E_F^{hyb} + E_R^{hyb}} \tag{7}$$

$$\chi_W^{hyb} = \frac{f^{hyb}_{\substack{F\\W}}}{f^{hyb}_{\substack{R\\W}}} \tag{8}$$

[1] In the case of solar energy, as further discussed in our case study in Section 4, P_R^{hyb} can be taken to be the primary energy associated with the incident radiation, but it could also be for example the primary fossil energy that some reference heating facility would consume to provide the same heat that the mirrors deliver to the plant, at the same temperature conditions. Of course, for consistency, one should adopt the corresponding choice also for the primary energy P_R^{sr} used by non-hybrid solar power plants that use the same mirror technology.

[2] The "primary energy factor" of a given product is the amount of primary energy consumed to produce a unit of that product, taking into account all processes in its life cycle. In the case of a power plant, it equals the inverse of the conversion efficiency calculated on the basis of the overall primary energy consumption, i.e., not just the direct consumption in the facility itself but also in all the processes in its life cycle (for example, for natural gas Ref. [19]. suggests considering the actual consumption incremented by 10% to account for the energy consumed during extraction, purification and transportation of the fuel).

[3] For solar energy, if we take the energy of the incident radiation intercepted by the mirrors as the primary energy, like we do in Section 4 for our case study, then we have $E_R^{hyb} = P_R^{hyb}$; if we take the energy of the incident radiation intercepted by the field where the mirrors are placed, then E_R^{hyb}/P_R^{hyb} is the solar field efficiency; if we take the primary fossil energy that some reference heating facility would consume to provide the same heat that the mirrors deliver to the plant, then E_R^{hyb}/P_R^{hyb} is yet another efficiency.

It is noteworthy that the parameter η_{hyb} is the overall energy conversion efficiency of the hybrid plant. It is important to note that in this paper we consider only allocation approaches whereby a given hybrid plant is not characterized in terms of details of its integration technology, but only by its overall efficiency η_{hyb}, the ratio σ_{hyb} of the primary resources it uses, and the ratio $\chi_{\mathrm{res}}^{\mathrm{hyb}}$ of their respective primary energy factors. As a result, the average primary energy factor of the mix of resources used by the hybrid plant can be written as follows

$$f_{\mathrm{hyb}} = \frac{P_{\mathrm{F}}^{\mathrm{hyb}} + P_{\mathrm{R}}^{\mathrm{hyb}}}{E_{\mathrm{F}}^{\mathrm{hyb}} + E_{\mathrm{R}}^{\mathrm{hyb}}} = \frac{f_{\mathrm{F}}^{\mathrm{hyb}} E_{\mathrm{F}}^{\mathrm{hyb}} + f_{\mathrm{R}}^{\mathrm{hyb}} E_{\mathrm{R}}^{\mathrm{hyb}}}{E_{\mathrm{F}}^{\mathrm{hyb}} + E_{\mathrm{R}}^{\mathrm{hyb}}} = \frac{\left(\sigma_{\mathrm{hyb}} + 1\right) f_{\mathrm{F}}^{\mathrm{hyb}} f_{\mathrm{R}}^{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} f_{\mathrm{R}}^{\mathrm{hyb}} + f_{\mathrm{F}}^{\mathrm{hyb}}} \tag{9}$$

A few substitutions and rearrangements allow us to write the relations between the allocation fractions and the partial primary energy factors in terms of the other parameters of the hybrid facility as follows

$$\beta_{\substack{\mathrm{W}\\\mathrm{R}}}^{\mathrm{hyb}} = \frac{\chi_{\mathrm{W}}^{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}} \quad \text{and} \quad \beta_{\substack{\mathrm{W}\\\mathrm{F}}}^{\mathrm{hyb}} = \frac{\sigma_{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}} \tag{10}$$

$$f_{\substack{\mathrm{R}\\\mathrm{W}}}^{\mathrm{hyb}} = \frac{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}\right) f_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + 1\right) \chi_{\mathrm{W}}^{\mathrm{hyb}} \eta_{\mathrm{hyb}}} \quad \text{and} \quad f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}} = \frac{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}\right) f_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + 1\right) \eta_{\mathrm{hyb}}} \tag{11}$$

$$\frac{P_{\mathrm{R}}^{\mathrm{hyb}}}{W_{\mathrm{hyb}}} = f_{\substack{\mathrm{R}\\\mathrm{W}}}^{\mathrm{hyb}} \beta_{\substack{\mathrm{W}\\\mathrm{R}}}^{\mathrm{hyb}} = \frac{f_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + 1\right) \eta_{\mathrm{hyb}}} \tag{12}$$

$$\frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{W_{\mathrm{hyb}}} = f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}} \beta_{\substack{\mathrm{W}\\\mathrm{F}}}^{\mathrm{hyb}} = \frac{\sigma_{\mathrm{hyb}} f_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + 1\right) \eta_{\mathrm{hyb}}} \tag{13}$$

These expressions show that the allocation problem is fully closed once a criterion to determine the value of $\chi_{\mathrm{W}}^{\mathrm{hyb}}$ is given. Such criterion characterizes the allocation method.

Finally we define and compute as follows the 'partial energy-conversion efficiencies' of the hybrid plant. The efficiency of conversion of fossil-fuel energy to electricity is

$$\eta_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}} = \frac{\beta_{\substack{\mathrm{W}\\\mathrm{F}}}^{\mathrm{hyb}} W_{\mathrm{hyb}}}{P_{\mathrm{F}}^{\mathrm{hyb}} / f_{\mathrm{F}}^{\mathrm{hyb}}} = \frac{f_{\mathrm{F}}^{\mathrm{hyb}}}{f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}}} = \frac{\left(\sigma_{\mathrm{hyb}} + 1\right) f_{\mathrm{F}}^{\mathrm{hyb}} \eta_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}\right) f_{\mathrm{hyb}}} \tag{14}$$

The efficiency of conversion of renewable resource energy to electricity is

$$\eta_{\substack{\mathrm{R}\\\mathrm{W}}}^{\mathrm{hyb}} = \frac{\beta_{\substack{\mathrm{W}\\\mathrm{R}}}^{\mathrm{hyb}} W_{\mathrm{hyb}}}{P_{\mathrm{R}}^{\mathrm{hyb}} / f_{\mathrm{R}}^{\mathrm{hyb}}} = \frac{f_{\mathrm{R}}^{\mathrm{hyb}}}{f_{\substack{\mathrm{R}\\\mathrm{W}}}^{\mathrm{hyb}}} = \frac{\left(\sigma_{\mathrm{hyb}} + 1\right) \chi_{\mathrm{W}}^{\mathrm{hyb}} f_{\mathrm{R}}^{\mathrm{hyb}} \eta_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{hyb}}\right) f_{\mathrm{hyb}}} \tag{15}$$

If the method adopted for allocation is "fair", then the above efficiencies represent an impartial apportionment of the benefits of hybridization among the different resources, thus allowing fair terms of comparison between the efficiencies of the hybrid facility and either the respective single-resource facilities in a local area, that we shall denote by $\eta_{\mathrm{FW}}^{\mathrm{sr}}$ and $\eta_{\mathrm{RW}}^{\mathrm{sr}}$ (where the superscript 'sr' stands for single resource), or some 'reference' partial energy-conversion efficiencies chosen by some local authority, that we shall denote by $\eta_{\mathrm{FW}}^{\mathrm{ref}}$ and $\eta_{\mathrm{RW}}^{\mathrm{ref}}$. The efficiencies $\eta_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}}$ and $\eta_{\substack{\mathrm{R}\\\mathrm{W}}}^{\mathrm{hyb}}$ are important parameters with which energy management regulators can devise fair incentive and taxation policies. In Section 5, we return on this topic while defining other related performance indicators.

In the remainder of this section we first briefly review the most commonly used allocation method considered so far for hybrid plants, including the exergy-based method ExRR. We then introduce the fixed-reference method SRSPR that extends to hybrid plants the same logic of fixed-reference and reversible-reference allocation, respectively, that characterizes the 'classical' methods of allocation most commonly considered for cogeneration plants (see Ref. [1]). For each of these methods, we show how it applies to the present allocation problem and we provide a brief critique which motivates the STALPR method we propose in Section 3.

2.1. Fossil-centered-solar-share (FCSS) allocation method

According to this allocation method, $f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}}$ is fixed to a reference value $f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{ref}}$ (normally assigned by some local authority for each type of fuel) representative of the inverse of the efficiency with which the same primary fossil-fuel resource is used for power production in a reference technology, for instance the best available technology or a representative average of non-hybrid (i.e., single-resource) technology. In other words, this allocation method assumes the closure relation

$$f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{hyb}} = f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{ref}} \tag{16}$$

From the solution of Eqs. (1)–(4) we obtain

$$\beta_{\substack{\mathrm{W}\\\mathrm{F}}}^{\mathrm{hyb}} = \frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{ref}} W_{\mathrm{hyb}}} = \frac{\sigma_{\mathrm{hyb}} f_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + 1\right) \eta_{\mathrm{hyb}} f_{\substack{\mathrm{F}\\\mathrm{W}}}^{\mathrm{ref}}} \tag{17}$$

meaning that the production of electrical energy allocated to the fossil fuel is the amount that would be produced with the reference technology by consuming the same amount of fuel primary energy $P_{\mathrm{F}}^{\mathrm{hyb}}$ consumed by the hybrid facility. The remaining portion of electricity is the "solar share"

$$\beta_{\substack{\mathrm{W}\\\mathrm{R}}}^{\mathrm{hyb}} = \frac{W_{\mathrm{hyb}} - \eta_{\mathrm{FW}}^{\mathrm{ref}} E_{\mathrm{F}}^{\mathrm{hyb}}}{W_{\mathrm{hyb}}} = 1 - \beta_{\substack{\mathrm{W}\\\mathrm{F}}}^{\mathrm{hyb}} \tag{18}$$

attributed to the renewable resource. This solar share coincides with that suggested for example in Ref. [15] and yields, according to Eq. (15), an expression of the (partial) efficiency of conversion of renewable resource energy to electricity

$$\eta_{\substack{\mathrm{R}\\\mathrm{W}}}^{\mathrm{hyb}} = \frac{W_{\mathrm{hyb}} - \eta_{\mathrm{FW}}^{\mathrm{ref}} E_{\mathrm{F}}^{\mathrm{hyb}}}{E_{\mathrm{R}}^{\mathrm{hyb}}} \tag{19}$$

that coincides with the "incremental solar efficiency" defined in Ref. [8].

From Eqs. (1)–(4) we also obtain the following expressions for χ_W^{hyb} and $f_{\substack{R\\W}}^{hyb}$ which are implied by the closure condition (16)

$$\chi_W^{hyb} = \frac{\left(\sigma_{hyb}+1\right)\eta_{hyb} f_{\substack{F\\W}}^{ref}}{f_{hyb}} - \sigma_{hyb} \tag{20}$$

$$f_{\substack{R\\W}}^{hyb} = \frac{f_{\substack{F\\W}}^{ref}}{\chi_W^{hyb}} \tag{21}$$

2.2. Exergy-based Reversible-Reference (ExRR) allocation method

To our knowledge an exergy allocation method has not yet been proposed in the context of source hybridization. It is, however, a straightforward extension of the conjugate allocation method that has been extensively discussed for cogeneration (see Ref. [1]). Its closure condition is

$$\chi_W^{hyb} = \chi_W^{Ex} \text{ with } \chi_W^{Ex} = \frac{f_{\substack{F\\W}}^{Ex}}{f_{\substack{R\\W}}^{Ex}} \tag{22}$$

where

$$f_{\substack{F\\W}}^{Ex} = \frac{P_F^{hyb}}{Ex_F^{hyb}} = \frac{P_F^{hyb}}{W_{F,ideal}^{hyb}} \text{ and } f_{\substack{R\\W}}^{Ex} = \frac{P_R^{hyb}}{Ex_R^{hyb}} = \frac{P_R^{hyb}}{W_{R,ideal}^{hyb}} \tag{23}$$

are the primary-energy-to-exergy ratios[4] of the fossil and the renewable primary resources used in the hybrid facility, $Ex_F^{hyb} = W_{F,ideal}^{hyb}$ represents the electricity that would be produced by any ideal reversible process that consumes the same amount of fuel as the hybrid plant, and $Ex_R^{hyb} = W_{R,ideal}^{hyb}$ the electricity that would be produced by any ideal reversible process that consumes the same amount of renewable resource as the hybrid plant.

By solving the system of Eqs. (1)–(4) and (22) we obtain

$$\beta_{\substack{W\\R}}^{hyb} = \frac{\dfrac{P_R^{hyb}}{f_{\substack{R\\W}}^{Ex}}}{\dfrac{P_R^{hyb}}{f_{\substack{R\\W}}^{Ex}} + \dfrac{P_F^{hyb}}{f_{\substack{F\\W}}^{Ex}}} \text{ and } \beta_{\substack{W\\F}}^{hyb} = \frac{\dfrac{P_F^{hyb}}{f_{\substack{F\\W}}^{Ex}}}{\dfrac{P_R^{hyb}}{f_{\substack{R\\W}}^{Ex}} + \dfrac{P_F^{hyb}}{f_{\substack{F\\W}}^{Ex}}} \tag{24}$$

or equivalently

$$\beta_{\substack{W\\R}}^{hyb} = \frac{Ex_R^{hyb}}{Ex_R^{hyb}+Ex_F^{hyb}} \text{ and } \beta_{\substack{W\\F}}^{hyb} = \frac{Ex_F^{hyb}}{Ex_R^{hyb}+Ex_F^{hyb}} \tag{25}$$

Thus, the allocation fractions are based on the relative proportions of the exergies of the resources consumed by the hybrid plant, i.e., on the relative proportions of the electricity that would be produced from the two resources in a hypothetical reference scenario in which every machinery operates reversibly.

The relations between the allocation fractions, the partial primary energy factors, and the other parameters of the hybrid facility, can be written according to Eqs. (9) and (10) as follows

$$\beta_{\substack{W\\R}}^{hyb} = \frac{\chi_W^{Ex}}{\sigma_{hyb}+\chi_W^{Ex}} \text{ and } \beta_{\substack{W\\F}}^{hyb} = \frac{\sigma_{hyb}}{\sigma_{hyb}+\chi_W^{Ex}} \tag{26}$$

$$f_{\substack{R\\W}}^{hyb} = \frac{\left(\sigma_{hyb}+\chi_W^{Ex}\right)f_{hyb}}{\left(\sigma_{hyb}+1\right)\chi_W^{Ex}\eta_{hyb}} \text{ and } f_{\substack{F\\W}}^{hyb} = \frac{\left(\sigma_{hyb}+\chi_W^{Ex}\right)f_{hyb}}{\left(\sigma_{hyb}+1\right)\eta_{hyb}} \tag{27}$$

We note that other allocation methods have been considered in the literature. For example, one based on the simple proportion of energy input [16], assigns the solar share as

$$\beta_{\substack{W\\R}}^{hyb} = \frac{E_R^{hyb}}{E_F^{hyb}+E_R^{hyb}} \tag{28}$$

Although conceptually criticizable because it considers fossil and renewable energy with the same 'weight', it is noteworthy that for the case at hand, namely, for hybrid power-only production from hydrocarbon fuels and solar radiation, Eq. (28) gives almost the same results as the exergy method, Eq. (25). This is because for most hydrocarbons $Ex_F^{hyb} \approx E_F^{hyb}$ to within ±5% (see, e.g., Ref. [20]), and for solar radiation $Ex_R^{hyb} \approx 0.93 E_R^{hyb}$ (see, e.g., Refs. [21,22], and references therein for discussions about the exergy of solar radiation).

2.3. Single-Resource-Separate-Production-Reference (SRSPR) allocation method

To our knowledge, the allocation method we propose in this section has not yet been suggested in the context of source hybridization. However, it too is a straightforward extension of the conjugate SPR allocation method that has been extensively adopted by regulators in the context of cogeneration (see Ref. [1]). This method assumes the following closure condition

$$\chi_W^{hyb} = \chi_W^{ref} \text{ with } \chi_W^{ref} = \frac{f_{\substack{F\\W}}^{ref}}{f_{\substack{R\\W}}^{ref}} \tag{29}$$

where $f_{\substack{R\\W}}^{ref}$ and $f_{\substack{F\\W}}^{ref}$ are reference partial primary energy factors chosen by some authority as representative of the performance of the (best available or representative average, usually but not necessarily single-resource) power production technologies that use, respectively, the same renewable resource and the same fossil fuel as the hybrid facility.

Combining Eqs. (1)–(4) and (29) we obtain

$$\beta_{\substack{W\\R}}^{hyb} = \frac{\dfrac{P_R^{hyb}}{f_{\substack{R\\W}}^{ref}}}{\dfrac{P_R^{hyb}}{f_{\substack{R\\W}}^{ref}} + \dfrac{P_F^{hyb}}{f_{\substack{F\\W}}^{ref}}} \text{ and } \beta_{\substack{W\\F}}^{hyb} = \frac{\dfrac{P_F^{hyb}}{f_{\substack{F\\W}}^{ref}}}{\dfrac{P_R^{hyb}}{f_{\substack{R\\W}}^{ref}} + \dfrac{P_F^{hyb}}{f_{\substack{F\\W}}^{ref}}} \tag{30}$$

Notably the ratios $P_R^{hyb}/f_{\substack{R\\W}}^{ref}$ and $P_F^{hyb}/f_{\substack{F\\W}}^{ref}$ represent respectively the amounts of electricity that would be produced in reference single-

[4] The primary-energy-to-exergy ratio of a given resource is defined as the ratio of the primary energy associated with a unit of that resource to the exergy per unit resource.

resource (renewable-only and fossil-fuel only) power plants consuming the same amount of fossil and renewable primary energy as the hybrid facility. In other words, the allocation fractions $\beta^{\rm hyb}_{\rm W\atop R}$ and $\beta^{\rm hyb}_{\rm W\atop F}$ are based on the relative proportions of the electricity that would be produced with the same primary energy consumptions in non-hybrid facilities operating with the reference partial primary energy factors $f^{\rm ref}_{\rm R\atop W}$ and $f^{\rm ref}_{\rm F\atop W}$, respectively.

The relations between the allocation fractions, the partial primary energy factors, and other parameters of the hybrid facility, according to Eqs. (9) and (10) are as follows

$$\beta^{\rm hyb}_{\rm W\atop R} = \frac{\chi^{\rm ref}_{\rm W}}{\sigma_{\rm hyb}+\chi^{\rm ref}_{\rm W}} \quad \text{and} \quad \beta^{\rm hyb}_{\rm W\atop F} = \frac{\sigma_{\rm hyb}}{\sigma_{\rm hyb}+\chi^{\rm ref}_{\rm W}} \tag{31}$$

$$f^{\rm hyb}_{\rm R\atop W} = \frac{\left(\sigma_{\rm hyb}+\chi^{\rm ref}_{\rm W}\right)f_{\rm hyb}}{\left(\sigma_{\rm hyb}+1\right)\chi^{\rm ref}_{\rm W}\eta_{\rm hyb}} \quad \text{and} \quad f^{\rm hyb}_{\rm F\atop W} = \frac{\left(\sigma_{\rm hyb}+\chi^{\rm ref}_{\rm W}\right)f_{\rm hyb}}{\left(\sigma_{\rm hyb}+1\right)\eta_{\rm hyb}} \tag{32}$$

2.4. *Critique of the FCSS, ExRR, and SRSPR allocation methods*

The problem with the FCSS method is that while the definition of the annual energy balance of the hybrid plant requires the three quantities $W_{\rm hyb}$, $P^{\rm hyb}_{\rm F}$, $P^{\rm hyb}_{\rm R}$, only the first two are used in the allocation, as apparent from Eq. (17). This leads to the inconsistency that two fossil-solar hybrid plants with the same value of $P^{\rm hyb}_{\rm F}$ and $W_{\rm hyb}$, would result in the same fraction of solar electricity, regardless of the size (land occupation and mirror area) of the solar field. Moreover, the benefit of hybridization, i.e., the incremental electricity production with respect to a reference non-hybrid facility that uses the same amount of fossil fuel, is assigned by the FCSS method entirely to the renewable source, thus overestimating the partial solar-to-electricity conversion efficiency which in turn may distort the local energy market.

To better explain this problem, we consider a hybrid plant with the following yearly energy balance: $P^{\rm hyb}_{\rm F} = 1878$ GWh, $P^{\rm hyb}_{\rm R} = 425$ GWh, and $W_{\rm hyb} = 867$ GWh, and let us assume $f^{\rm hyb}_{\rm F} = 1.1$ (natural gas) and $f^{\rm ref}_{\rm F\atop W} = 2.895$ (or equivalently $\eta^{\rm ref}_{\rm FW} = 0.38$) so that $E^{\rm hyb}_{\rm F} = 1878/1.1 = 1707$, and $f^{\rm hyb}_{\rm R} = 1$ so that $E^{\rm hyb}_{\rm R} = 425$. That these figures are representative of a realistic hybrid facility will become apparent in Section 4. Eq. (18) yields $\beta^{\rm hyb}_{\rm W\atop R} = 1 - 0.38 \times 1707/867 = 0.252$ and through Eq. (19) or equivalently Eq. (15) $\eta^{\rm hyb}_{\rm R\atop W} = (867-0.38 \times 1707)/425 = 0.513$, which is a clearly unrealistically high figure, while through Eq. (14) we obtain $\eta^{\rm ref}_{\rm FW} = 0.38$, confirming that the method grants no fraction of the benefits from hybridization to the fossil resource.

Both the ExRR and the SRSPR methods overcome this problem, by taking into account the contributions of both primary resources in the computation of the allocation problem.

The ExRR method bases the allocation on the primary energy factors of the renewable and fossil sources that would characterize a hypothetical reference scenario of thermodynamically reversible production of electricity.

Let us illustrate again this idea for the case of the hybrid plant just considered. By computing the fossil and renewable exergies, respectively, as $\mathrm{Ex}^{\rm hyb}_{\rm F} = P^{\rm hyb}_{\rm F} = 1878$ GWh and $\mathrm{Ex}^{\rm hyb}_{\rm R} = 0.93P^{\rm hyb}_{\rm R} = 395$ GWh, through Eq. (25) we obtain $\beta^{\rm hyb}_{\rm W\atop R} = 395/(395+1878) = 0.174$ and $\beta^{\rm hyb}_{\rm W\atop F} = 0.826$. The resulting efficiencies are, therefore, through Eq. (15) $\eta^{\rm hyb}_{\rm R\atop W} = 0.174 \times 867/425 = 0.355$ and through Eq. (14) $\eta^{\rm hyb}_{\rm F\atop W} = 0.826 \times 867/1707 = 0.42$, confirming the unrealistically high value for the renewable source. The reason for the overestimate of the solar efficiency is that current technologies of power production from fossil fuels are much closer to ideal (i.e., to 100% second-law efficiency) than those from renewables. So, though based on sound thermodynamic concepts, the ExRR method, if adopted as a basis of regulations, would grant an unfairly high advantage to the renewable resource, at least as long as renewable technologies maintain a level of second-law efficiency lower than fossil-fuel technologies.

The SRSPR method partly resolves this problem by providing more realistic figures whenever the adopted reference primary energy factors reflect the state of the art technology of the fuel to electricity conversion as well as of solar to electricity technology. In fact, if in the previous example we assume $f^{\rm ref}_{\rm R\atop W} = 6.54$ and $f^{\rm ref}_{\rm F\atop W} = 2.895$ (equivalent to $\eta^{\rm ref}_{\rm RW} = 0.153$ and $\eta^{\rm ref}_{\rm FW} = 0.38$) we obtain, by substituting into Eq. (30), $\beta^{\rm hyb}_{\rm W\atop R} = 425/6.54/(425/6.54 + 1878/2.895) = 0.091$ and $\beta^{\rm hyb}_{\rm W\atop F} = 0.909$ which in turn, through Eq. (15) yields $\eta^{\rm hyb}_{\rm R\atop W} = 0.091 \times 867/425 = 0.186$ and through Eq. (14) $\eta^{\rm hyb}_{\rm F\atop W} = 0.909 \times 867/1707 = 0.462$. These are obviously much more realistic figures and show that the SRSPR method provides a better share-out of the benefits of hybridization between the two sources.

Nevertheless, we wish to emphasize that also the SRSPR method has fundamental drawbacks. The first is that it is based on assigned reference efficiencies that may differ from the actual values which characterize the energy conversion portfolio of the local area where the hybrid facility is to be installed. The second is that the reference values, being fixed by some authority, are not dynamically influenced by the installation of new hybrid facilities, and therefore the SRSPR method neglects the effects associated with the modification of the local energy portfolio. This fact may result in distortions of the local energy market, unless the authority continuously updates the reference efficiencies by taking into account the progressive penetration of hybridization. In the latter case, the fundamental drawbacks of the SPSPR method are overcome and the method effectively becomes equivalent to the STALPR method we propose in the next section.

3. STALPR allocation method for a hybrid fossil-and-solar power plant

Similar to what we observed in our previous paper [1] on cogeneration, a limitation of the classical allocation criteria is that they are based on some prescribed reference efficiencies for each resource to product conversion occurring in the hybrid facility assigned by some authority and in general differ from the actual average ratios that characterize the local energy portfolio. To overcome this problem for the case of cogeneration, we proposed a self-consistent method whereby the allocation is adaptive and self-tuned to the local energy production portfolio. We call it the Self-Tuned Average-Local-Productions Reference (STALPR) method. In this section, we extend the method to the case of a fossil/renewable hybrid power plant operating in a local area where all the other electricity power facilities are single-resource, n using fossil fuels, and m using renewable resources. Localizing the allocations is particularly important in this case since the performance of renewable energy facilities depends strongly on the local conditions.

We assume for simplicity that both the set of n fossil plants and the set of m renewable plants are conceptually lumped together into single overall units producing respectively the overall amounts of electrical energy $W_{\mathrm{R}}^{\mathrm{sr}} = \sum_{i=1}^{n} W_{\mathrm{R}}^{\mathrm{sr},i}$ and $W_{\mathrm{F}}^{\mathrm{sr}} = \sum_{i=1}^{m} W_{\mathrm{F}}^{\mathrm{sr},i}$, and characterized by the average renewable and fossil primary energy factor

$$\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{R}\\\mathrm{W}}} = \frac{\sum_{i=1}^{n} f^{\mathrm{sr},i}_{\substack{\mathrm{R}\\\mathrm{W}}} W_{\mathrm{R}}^{\mathrm{sr},i}}{\sum_{i=1}^{n} W_{\mathrm{R}}^{\mathrm{sr},i}} \quad \text{and} \quad \bar{f}^{\mathrm{sr}}_{\substack{\mathrm{F}\\\mathrm{W}}} = \frac{\sum_{i=1}^{m} f^{\mathrm{sr},i}_{\substack{\mathrm{F}\\\mathrm{W}}} W_{\mathrm{F}}^{\mathrm{sr},i}}{\sum_{i=1}^{m} W_{\mathrm{F}}^{\mathrm{sr},i}}$$

A sketch of the local-area powerplant scenario is shown in Fig. 2.

The rationale of the proposed method is that the allocation parameters should not be based on static reference values, but should be self-determined by the method itself as characteristic average features of the actual energy production scenario and mix of resources used in the local area of interest, including the hybrid facility. Therefore, to characterize the local-area scenario we define $f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}$ and $f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}$ as the average primary energy factors for the fossil fuel and renewable resource conversion to electricity in the local area. With reference to Fig. 2 they are calculated as

$$f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}} = \frac{\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{R}\\\mathrm{W}}} W_{\mathrm{R}}^{\mathrm{sr}} + f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\\mathrm{W}}} W_{\mathrm{R}}^{\mathrm{hyb}}}{W_{\mathrm{R}}^{\mathrm{sr}} + W_{\mathrm{R}}^{\mathrm{hyb}}} \quad \text{and} \quad f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}} = \frac{\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{F}\\\mathrm{W}}} W_{\mathrm{F}}^{\mathrm{sr}} + f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\\mathrm{W}}} W_{\mathrm{F}}^{\mathrm{hyb}}}{W_{\mathrm{F}}^{\mathrm{sr}} + W_{\mathrm{F}}^{\mathrm{hyb}}} \tag{33}$$

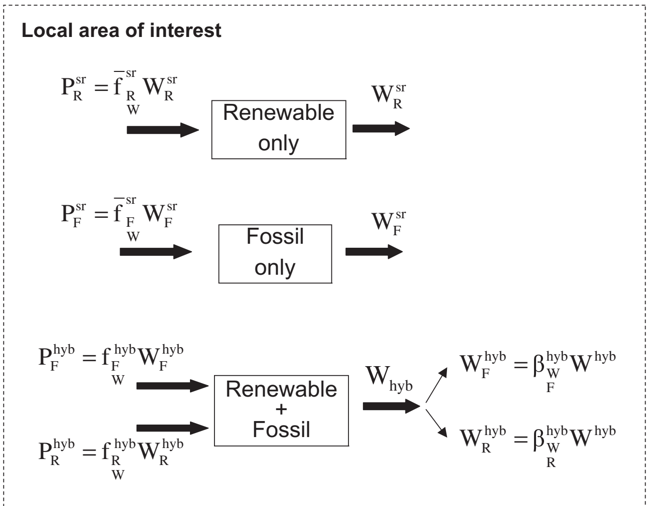

Fig. 2. Schematic representation of a local area of interest with a single hybrid plant.

or equivalently, substituting the definition of the β's given in Eq. (1)

$$f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}} = \frac{\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{R}\\\mathrm{W}}} W_{\mathrm{R}}^{\mathrm{sr}} + f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\\mathrm{W}}} \beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{R}}} W_{\mathrm{hyb}}}{W_{\mathrm{R}}^{\mathrm{sr}} + \beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{R}}} W_{\mathrm{hyb}}} \quad \text{and} \quad f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}} = \frac{\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{F}\\\mathrm{W}}} W_{\mathrm{F}}^{\mathrm{sr}} + f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\\mathrm{W}}} \beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{F}}} W_{\mathrm{hyb}}}{W_{\mathrm{F}}^{\mathrm{sr}} + \beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{F}}} W_{\mathrm{hyb}}} \tag{34}$$

Therefore, these factors are assigned according to the existing plants in the local area.[5] Then, following the same logic adopted for the classical SRSPR method, we close the allocation problem by adopting the following rule

$$\chi_{\mathrm{W}}^{\mathrm{hyb}} = \chi_{\mathrm{W}}^{\mathrm{loc}} \quad \text{with} \quad \chi_{\mathrm{W}}^{\mathrm{loc}} = \frac{f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}}{f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}} \tag{35}$$

so that combining Eqs. (1)–(4) and (35) we obtain

$$\beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{R}}} = \frac{\dfrac{P_{\mathrm{R}}^{\mathrm{hyb}}}{f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}}}{\dfrac{P_{\mathrm{R}}^{\mathrm{hyb}}}{f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}} + \dfrac{P_{\mathrm{F}}^{\mathrm{hyb}}}{f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}}} \quad \text{and} \quad \beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{F}}} = \frac{\dfrac{P_{\mathrm{F}}^{\mathrm{hyb}}}{f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}}}{\dfrac{P_{\mathrm{R}}^{\mathrm{hyb}}}{f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}} + \dfrac{P_{\mathrm{F}}^{\mathrm{hyb}}}{f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}}} \tag{36}$$

meaning that the allocation fractions β's are based on the relative proportions of the electrical energies that would be produced in single-resource facilities consuming the same primary resources as the hybrid facility but operating according to the average local-area primary energy factors $f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}$ and $f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}$ of the respective resources.

The system of ten Eqs. (1)–(4), (34), (35) and (36) can be solved for the ten unknowns $\beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{R}}}$, $\beta^{\mathrm{hyb}}_{\substack{\mathrm{W}\\\mathrm{F}}}$, $P_{\mathrm{R}}^{\mathrm{hyb}}$, $P_{\mathrm{F}}^{\mathrm{hyb}}$, $f^{\mathrm{hyb}}_{\substack{\mathrm{R}\\\mathrm{W}}}$, $f^{\mathrm{hyb}}_{\substack{\mathrm{F}\\\mathrm{W}}}$, $W_{\mathrm{R}}^{\mathrm{hyb}}$, $W_{\mathrm{F}}^{\mathrm{hyb}}$, $f^{\mathrm{loc}}_{\substack{\mathrm{R}\\\mathrm{W}}}$ and $f^{\mathrm{loc}}_{\substack{\mathrm{F}\\\mathrm{W}}}$ in terms of given values for W_{hyb}, $E_{\mathrm{R}}^{\mathrm{hyb}}$, $E_{\mathrm{F}}^{\mathrm{hyb}}$, $f_{\mathrm{R}}^{\mathrm{hyb}}$, $f_{\mathrm{F}}^{\mathrm{hyb}}$, $\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{R}\\\mathrm{W}}}$, $\bar{f}^{\mathrm{sr}}_{\substack{\mathrm{F}\\\mathrm{W}}}$, $W_{\mathrm{R}}^{\mathrm{sr}}$, $W_{\mathrm{F}}^{\mathrm{sr}}$. In general it can be solved numerically. The problem can further be reduced to the solution of a single second-order equation, therefore yielding an analytical solution. The derivation of the second-order equation is given in Appendix A. Also, the analysis of how the value of $\chi_{\mathrm{W}}^{\mathrm{loc}}$ defined in Eq. (35) depends on the various parameters of the problem can be readily conducted using the same methodology we used in the analysis of the corresponding Eq. (30) of Ref. [1]. Details are given in Appendix B.

As a result of such analysis, four typical cases may be identified according to the whether σ_{hyb} is smaller or greater than σ_{loc} and whether the Incremental Electricity Index (IEI) defined as follows

[5] It can be shown that if in the local area there are no 'purely' (i.e., single-resource) renewable plants, then the method reduces to the FCSS method. The method is instead undetermined if the local area contains neither purely renewable plants nor purely fossil plants, because then it contains only the hybrid plant and there are no local terms of comparison. The SRSPR method can be viewed as addressing such case, since the allocation is based on reference terms of comparisons that are independent of the actual local energy portfolio.

$$\mathrm{IEI_{hyb}} = \frac{W_{\mathrm{hyb}}}{\frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{f_{\mathrm{F\,W}}^{\mathrm{sr}}} + \frac{P_{\mathrm{R}}^{\mathrm{hyb}}}{f_{\mathrm{R\,W}}^{\mathrm{sr}}}} - 1 = \frac{\left(\sigma_{\mathrm{hyb}} f_{\mathrm{R}}^{\mathrm{hyb}} + f_{\mathrm{F}}^{\mathrm{hyb}}\right) f_{\mathrm{F}}^{\mathrm{sr}} f_{\mathrm{R}}^{\mathrm{sr}} \eta_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} f_{\mathrm{R}}^{\mathrm{sr}} \eta_{\mathrm{FW}}^{\mathrm{sr}} + f_{\mathrm{F}}^{\mathrm{sr}} \eta_{\mathrm{RW}}^{\mathrm{sr}}\right) f_{\mathrm{F}}^{\mathrm{hyb}} f_{\mathrm{R}}^{\mathrm{hyb}}} - 1 \tag{37}$$

is greater or smaller than zero. Clearly, $1 + \mathrm{IEI_{hyb}}$ represents the ratio of the electrical energy produced by the hybrid facility and the electrical energy that the single-resource facilities in the local area would produce with the same resources. It is noteworthy that:

- the case $\mathrm{IEI_{hyb}} > 0$ implies that the overall primary energy is converted with a higher efficiency in the hybrid plant than in the single-resource facilities;
- the case $\sigma_{\mathrm{hyb}} < \sigma_{\mathrm{loc}}$ represents a hybrid plant with a higher share of renewable energy than the average share in the local area.

4. Case study

In order to examine the features of the STLAPR approach, in this section we provide an example of its application to the renewable-to-fossil allocation of the electricity produced in hybrid single-product facilities, and we study the difference between the SRSPR and the STLAPR allocation methods as a function of:

(i) the degree of penetration of hybrid facilities in the given local area;
(ii) the reference efficiency of non-hybrid power production from fossil fuels;
(iii) the reference efficiency of non-hybrid power production from solar radiation;
(iv) the degree of penetration of non-hybrid power production from solar radiation in the local area;

For this purpose, we consider the local area shown in Fig. 2 and we assume that the annual demand of electrical energy is initially supplied according to the following shares:

- 90% by fossil-only (i.e., natural gas single-resource) power plants operating with $f_{\mathrm{F\,W}}^{\mathrm{sr}} = f_{\mathrm{F}}^{\mathrm{sr}}/\eta_{\mathrm{RW}}^{\mathrm{sr}}$, assuming an overall (yearly) average efficiency $\eta_{\mathrm{FW}}^{\mathrm{sr}} = 0.38$ and a primary energy factor $f_{\mathrm{F}}^{\mathrm{sr}} = 1.1$ (natural gas); for the purposes of the SRSPR method, we shall assume $\eta_{\mathrm{FW}}^{\mathrm{ref}} = \eta_{\mathrm{FW}}^{\mathrm{sr}}$;
- 10% by renewable-only (i.e., single-resource, non-hybrid solar) power plants operating with $f_{\mathrm{R\,W}}^{\mathrm{sr}} = f_{\mathrm{R}}^{\mathrm{sr}}/\eta_{\mathrm{RW}}^{\mathrm{sr}}$, assuming an overall (yearly) average efficiency $\eta_{\mathrm{RW}}^{\mathrm{sr}} = 0.153$ and a primary energy factor $f_{\mathrm{R}}^{\mathrm{sr}} = 1$; for the purposes of the SRSPR method, we shall assume $\eta_{\mathrm{RW}}^{\mathrm{ref}} = \eta_{\mathrm{RW}}^{\mathrm{sr}}$.

Next, we assume that the fossil-only facilities in the local area are progressively replaced by renewable-fossil hybrid plants, until all the single-resource fossil electricity $W_{\mathrm{F}}^{\mathrm{sr}}$ is eventually replaced by W_{hyb}, while the single-resource solar electricity $W_{\mathrm{R}}^{\mathrm{sr}}$ remains fixed to its initial value.

For simplicity in this case study, all renewable-fossil hybrid plants are assumed to be identical and all based on the technology of the Solar-Integrated Combined-Cycle System (SICCS) with parabolic trough solar field, as described in Ref. [12]. In this solution, the integration is achieved by boiling a fraction of the feed-water in the solar boiler and feeding saturated high-pressure steam to the main steam circuit of the combined cycle at the inlet of the superheater in the HRSG. It is therefore assumed that it is always possible to add a solar field in the area nearby the existing fossil-only facilities so as to convert them into SICCS.

To compute the energy allocation according to the STALPR method it is necessary to determine the global yearly energy balance on the hybrid power plant and to evaluate the parameters $P_{\mathrm{F}}^{\mathrm{hyb}}/W_{\mathrm{hyb}}$ and $P_{\mathrm{R}}^{\mathrm{hyb}}/W_{\mathrm{hyb}}$ which characterize the hybrid facility. These are obtained as follows from the data reported in Ref. [12].

- The annual electricity generation W_{hyb} is 867 GWh.
- $P_{\mathrm{F}}^{\mathrm{hyb}}$ is not given explicitly, but can be obtained from the value of the annual CO_2 emissions, recalling that (in the hypothesis that the fossil fuel is all CH_4), the combustion of 1 kg of CH_4 yields 2.75 kg of CO_2. Given that in this example 363,000 ton of CO_2 are emitted by the SICCS, we can readily obtain a corresponding fuel consumption of 122,900 ton of CH_4. Considering $\mathrm{LHV_{CH4}} = 50$ MJ/kg and assuming $f_{\mathrm{F}}^{\mathrm{hyb}} = 1.1$ we find a primary fossil energy consumption $P_{\mathrm{F}}^{\mathrm{hyb}} = 1878$ GWh.
- $P_{\mathrm{R}}^{\mathrm{hyb}}$ can be obtained from the design thermal capacity of the solar field which in this case is 90 MW. Assuming a parabolic trough solar field efficiency[6] of 66% and a design Direct Normal Insolation (DNI) of 800 W/m^2 [17], the resulting solar collectors area is 169,900 m^2, while the total solar field required land area[7] is 414,000 m^2. Assuming an annual insolation of 2500 kWh/year, characteristic of locations with favorable insolation conditions [18], the resulting solar energy falling onto the field is 1035 GWh, but that intercepted by the mirrors is 425 GWh. As typically done when computing efficiencies of solar power plants, we consider the latter to be the primary energy consumption and, therefore, we set $P_{\mathrm{R}}^{\mathrm{hyb}} = 425$ and the primary energy factor $f_{\mathrm{R}}^{\mathrm{hyb}} = 1$.[8] Since both the SRSPR and the STLAPR allocation models take as input data the overall annual balances on the hybrid plant, $P_{\mathrm{R}}^{\mathrm{hyb}}$ represents the annual reference insolation value that takes into account the hourly variable weather conditions of the site considered.

Table 1 summarizes the above assumptions and the other parameters of the case study. Notice that Eq. (37) yields IEI = 0.2146. Table 2 compares the FCSS, SRSPR, and ExRR methods, by providing the corresponding allocation fractions, efficiencies, and primary energy factors for the case study. Figs. 3 and 4 compare the SRSPR and STALPR methods by showing the allocation fractions and the efficiencies as functions of the "penetration of hybrid facilities in the local area" characterized by the following parameter:

$$x = \frac{W_{\mathrm{hyb}}}{W_{\mathrm{F}}^{\mathrm{sr}} + W_{\mathrm{hyb}}} \tag{38}$$

which for the local area represents the fraction of the electricity not produced in single-resource solar facilities that is produced in

[6] The efficiency of the solar field is defined as the ratio of the thermal power provided by the solar field to the CC to the insolation on the solar collectors area at design conditions. It is the product of optical, thermal, and piping efficiencies, assumed respectively equal to 74.4%, 90.0%, and 98.9% [17].

[7] The total solar field area includes a minimum distance between each row of parabolic trough collectors (usually in the range of 15 m) to limit shading [17].

[8] Alternatively, we could assume $P_{\mathrm{R}}^{\mathrm{hyb}} = 1035$ GWh and a primary energy factor $f_{\mathrm{R}}^{\mathrm{hyb}} = 1035/425$, i.e., equal to the ratio of the overall solar field area to the overall mirror area. However, in such case we should also set, for consistency, $f_{\mathrm{R}}^{\mathrm{sr}} = 1035/425$, because also the single-resource, non-hybrid solar power plants are assumed to use the same mirror technology as the hybrid plant. As a result, this alternate choice would not alter any of the results shown in Figs. 3–6 (neither for the SRSPR nor for the STALPR method).

Table 1
Summary of assumptions made and values of some important parameters for the hybrid solar-and-fossil power plant case study (in parenthesis the case with $\eta^{sr}_{\substack{F\\W}} = 0.50$).

Parameters of the local area	
Fraction of the renewable-only electricity in the local area, $\gamma^{sr,RW}_{W}$	0.1
Primary energy factor of solar energy, $f^{sr,RW}_{R}$	1.0
Average efficiency of the renewable-only facilities, $\eta^{sr}_{\substack{R\\W}}$	0.153
Primary energy factor of natural gas (assumed the only fuel used in the local area for fossil-only facilities), $f^{sr,FW}_{F}$	1.1
Average efficiency of the fossil-only facilities, $\eta^{sr}_{\substack{F\\W}}$	0.38 (0.50)
Ratio of primary energy factors of electricity produced in solar-only and fossil-only facilities, Eq. (43), χ^{sr}_{W}	0.4429 (0.3366)
Parameters of the hybrid facilities	
Primary energy factor of solar energy, f^{hyb}_{R}	1.0
Primary energy factor of natural gas (assumed the only fuel used in the local area for hybrid facilities), f^{hyb}_{F}	1.1
Renewable primary energy to electricity ratio, P^{hyb}_{R}/W_{hyb}	425/867
Fossil primary energy to electricity ratio, P^{hyb}_{F}/W_{hyb}	1878/867
Fossil to renewable primary energy ratio, $\sigma_{hyb} = P^{hyb}_{F}/P^{hyb}_{R}$	1878/425
Ratio of primary energy factors of solar energy and fossil fuel, Eq. (5), χ^{hyb}_{res}	1.1
Average primary energy factor of the resources, Eq. (9), f_{hyb}	1.08
Incremental Electricity Index, Eq. (37), IEI	0.2146 (−0.0562)

hybrid SICCS plants (for our case study it will range from 0 to 1). To compute the plots shown in Figs. 3 and 4, we define the following shorthand variables:

$$y = \beta^{hyb}_{\substack{W\\R}} \tag{39}$$

which is the allocation fraction assigned by the STALPR method to the fraction of the power production in hybrid facility that we should consider as obtained from the renewable resource;

$$g = \gamma^{sr}_{RW} \tag{40}$$

which is the fraction of the overall electricity produced in the local area that comes form the renewable-only facilities (for our case study $g = 0.1$);

$$h = \frac{f^{sr}_{\substack{F\\W}} W_{hyb} - P^{hyb}_{F}}{f^{sr}_{\substack{F\\W}} W_{hyb}} = 1 - \frac{\sigma_{hyb} f_{hyb}}{\left(\sigma_{hyb}+1\right) f^{sr}_{\substack{F\\W}}\ \eta_{hyb}} \tag{41}$$

which represents the index of fossil fuel savings obtained by the hybrid facility with respect to the fossil-only power plants in the area (in Section 5 this will be renamed as $\mathrm{PFES_{hyb}}$); and,

$$k = \frac{\dfrac{P^{hyb}_{R}}{f^{sr}_{\substack{R\\W}}}}{\dfrac{P^{hyb}_{R}}{f^{sr}_{\substack{R\\W}}} + \dfrac{P^{hyb}_{F}}{f^{sr}_{\substack{F\\W}}}} = \frac{\chi^{sr}_{W}}{\sigma_{hyb} + \chi^{sr}_{W}} \tag{42}$$

which represents the allocation fraction according to the SRSPR method based on reference values equal to the primary energy factors of the renewable-only and the fossil-only power plants in the area, i.e., as given from Eq. (30) with $f^{ref}_{\substack{R\\W}} = f^{sr}_{\substack{R\\W}}$ and $f^{ref}_{\substack{F\\W}} = f^{sr}_{\substack{F\\W}}$ or equivalently Eq. (31) with $\chi^{ref}_{W} = \chi^{sr}_{W}$ where

$$\chi^{sr}_{W} = \frac{f^{sr}_{\substack{F\\W}}}{f^{sr}_{\substack{R\\W}}} \tag{43}$$

In terms of the above parameters, it is easy to verify that the equations of the preceding section result in the following second-order equation in y

$$(k-g)xy^2 + [g - hkx(1-(1-g)x)]y - gk(1-hx) = 0 \tag{44}$$

which can be readily solved for the only positive root.

Figs. 3–5 show the results of the analysis for the case study defined by the parameters listed in Table 1. The sensitivity analysis shown in the figures is centered on the case study values of $\eta^{sr}_{RW} = 0.153$, $\eta^{sr}_{FW} = 0.38$, and $\gamma^{sr}_{RW} = 0.1$. It allows identifying how changes in the local area scenario affect the electricity allocation in the hybrid plant.

Results of the analysis are evaluated in terms of the renewable and fossil partial conversion efficiencies $\eta^{hyb}_{\substack{F\\W}}$ and $\eta^{hyb}_{\substack{R\\W}}$ and the allocation fractions $\beta^{hyb}_{\substack{W\\F}}$ and $\beta^{hyb}_{\substack{W\\R}}$ as function of the penetration of

Table 2
Summary of FCSS, SRSPR, and ExRR allocation results for the hybrid solar-fossil case study defined in Section 4 (parameters summarized in Table 1).

	$\eta^{ref}_{FW} = 0.38$, $\eta^{ref}_{RW} = 0.153$		$\eta^{ref}_{FW} = 0.50$, $\eta^{ref}_{RW} = 0.153$		$Ex_R/E_R = 0.93$
	FCSS	SRSPR	FCSS	SRSPR	ExRR
$\beta^{hyb}_{\substack{W\\F}}$	0.7483	0.9089	0.9846	0.9292	0.8261
$\beta^{hyb}_{\substack{W\\R}}$	0.2517	0.0911	0.0154	0.0708	0.1739
$\eta^{hyb}_{\substack{F\\W}}$	0.38	0.4616	0.50	0.4719	0.4195
$\eta^{hyb}_{\substack{R\\W}}$	0.5135	0.1858	0.0314	0.1444	0.3547
$f^{hyb}_{\substack{F\\W}}$	2.895	5.381	2.2	2.331	2.819
$f^{hyb}_{\substack{R\\W}}$	1.947	2.383	31.8	6.925	2.622
χ^{hyb}_{W}	1.486	0.4429	0.0692	0.3366	0.930

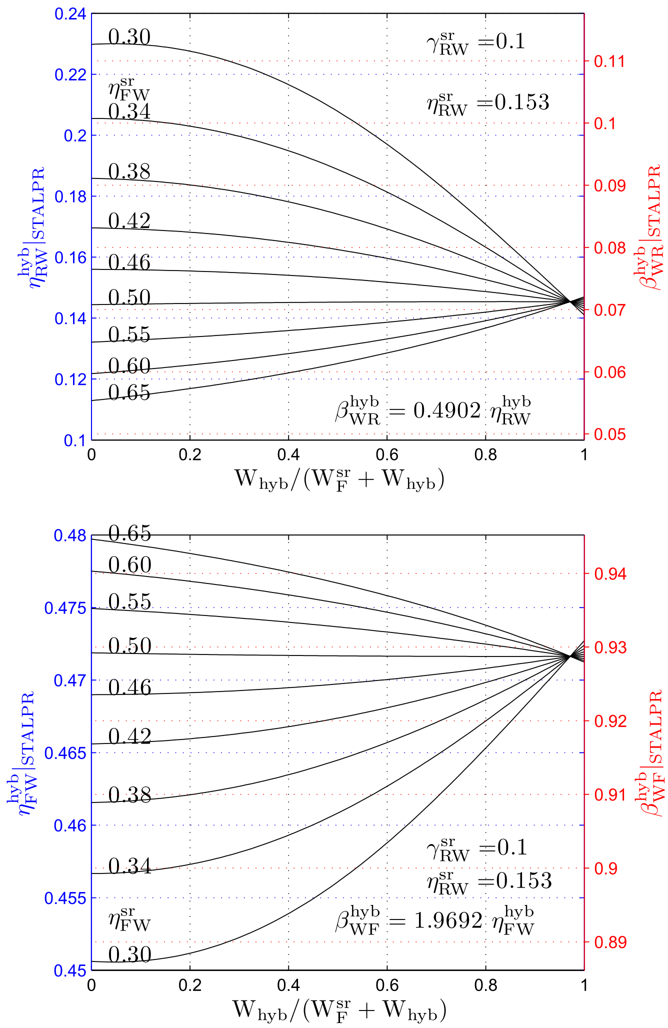

Fig. 3. Renewable-to-electricity and fossil-to-electricity efficiencies, $\eta^{hyb}_{\substack{W\\R}}$ and $\eta^{hyb}_{\substack{W\\F}}$, upper and lower graph respectively (left axes), and the corresponding renewable and fossil allocation fractions $\beta^{hyb}_{\substack{W\\R}}$ and $\beta^{hyb}_{\substack{W\\F}}$ (right axes) of the hybrid facility, plotted as functions of the hybrid penetration parameter $W_{hyb}/(W^{sr}_F + W_{hyb})$ for various values of η^{sr}_{FW}. The other parameters that define the hybrid plant and the local area are listed in Table 1.

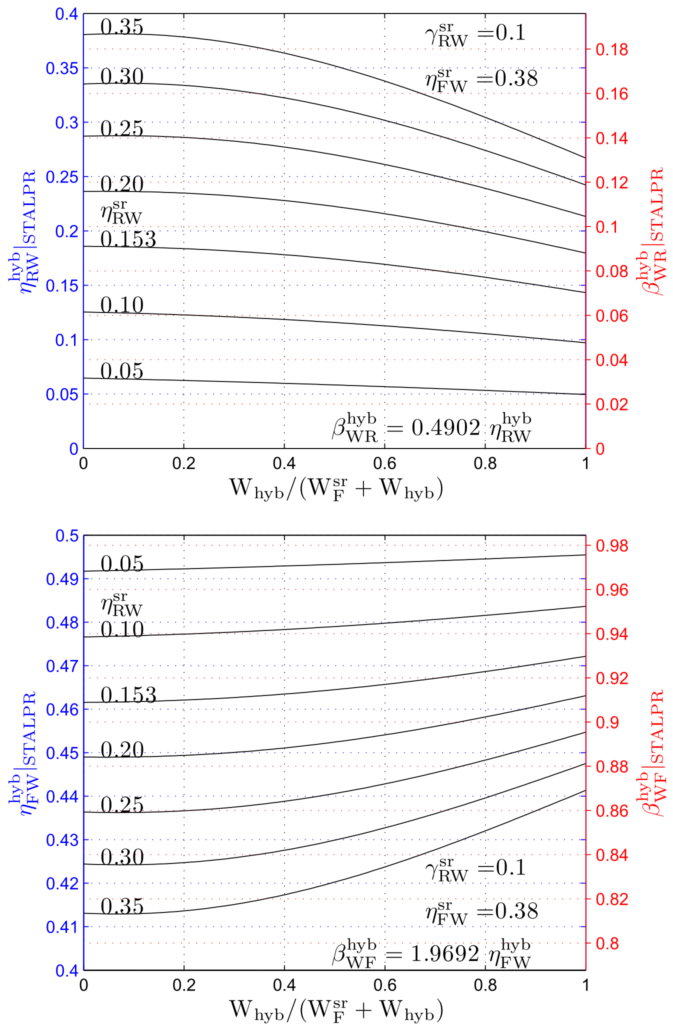

Fig. 4. Renewable-to-electricity and fossil-to-electricity efficiencies, $\eta^{hyb}_{\substack{W\\R}}$ and $\eta^{hyb}_{\substack{W\\F}}$, upper and lower graph respectively (left axes), and the corresponding renewable and fossil allocation fractions $\beta^{hyb}_{\substack{W\\R}}$ and $\beta^{hyb}_{\substack{W\\F}}$ (right axes) of the hybrid facility, plotted as functions of the hybrid penetration parameter $W_{hyb}/(W^{sr}_F + W_{hyb})$ for various values of η^{sr}_{RW}. The other parameters that define the hybrid plant and the local area are listed in Table 1.

hybrid facilities in the local area x. It is noteworthy that $\beta^{hyb}_{\substack{W\\F}}$ and $\beta^{hyb}_{\substack{W\\R}}$ are related to $\eta^{hyb}_{\substack{F\\W}}$ and $\eta^{hyb}_{\substack{R\\W}}$ by Eqs. (14) and (15), respectively, where the parameters W_{hyb}, P^{hyb}_F, P^{hyb}_R, f^{hyb}_F, f^{hyb}_R are constant in our case study. Thus the values of $\beta^{hyb}_{\substack{W\\F}}$ and $\eta^{hyb}_{\substack{F\\W}}$ can be represented by the same curve by adopting the correct scale ratio. The same holds as well for $\beta^{hyb}_{\substack{W\\R}}$ and $\eta^{hyb}_{\substack{R\\W}}$.

The curves in Fig. 3 converge into a single point located at $x < 1$. This indicates that for such particular value of x the allocation is independent of η^{sr}_{FW}. The existence of this point can be confirmed by solving for the positive root y admitted by Eq. (44) and calculating the value(s) of x for which the condition $\partial y/\partial \eta^{sr}_{FW} = 0$ is satisfied. This is a rather demanding task that is not carried out in view of the scarce impact on the general implications of the allocation model.

The same considerations are also valid for Figs. 4 and 5. In particular, for the values of the parameters of our case study, also the curves in Fig. 5 converge into a single point located at an $x < 1$ where the condition $\partial y/\partial \gamma^{sr}_{RW} = 0$ is satisfied indicating that for such particular value of x the allocation is independent of γ^{sr}_{RW}. It is clear from Fig. 4 that the condition $\partial y/\partial \eta^{sr}_{RW} = 0$ does not occur for an $x < 1$.

Clearly the limit values $x = 0$ and $x = 1$ represent, respectively, the condition of no hybrid plant in the local area and the case of complete conversion of all fossil-only electricity into solar-integrated hybrid. At $x = 0$ the STALPR method and the SRSPR method coincide provided that, for the latter, we assume as reference values the primary energy factors of the renewable-only and the fossil-only power plants in the local area, i.e., $\chi^{ref}_W = \chi^{sr}_W$.

The curves plotted in Fig. 3 refer to the sensitivity analysis on the parameter η^{sr}_{FW}, varied within the range 0.30–0.65. It can be observed that the slope of the curves that represent $\beta^{hyb}_{\substack{W\\R}}$ and $\eta^{hyb}_{\substack{R\\W}}$ is negative whenever, at $x = 0$, $\eta^{hyb}_{\substack{R\\W}} > \eta^{sr}_{RW}$, meaning a decrease in the

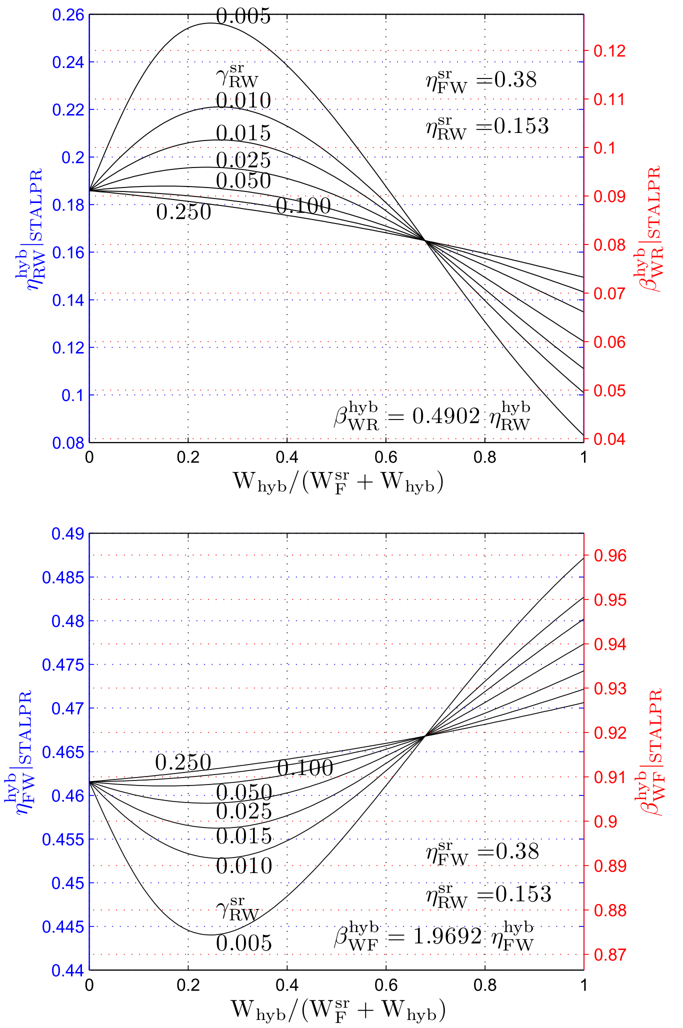

Fig. 5. Renewable-to-electricity and fossil-to-electricity efficiencies, $\eta^{\rm hyb}_{\rm RW}$ and $\eta^{\rm hyb}_{\rm FW}$, upper and lower graph respectively (left axes), and the corresponding renewable and fossil allocation fractions $\beta^{\rm hyb}_{\rm WR}$ and $\beta^{\rm hyb}_{\rm WF}$ (right axes) of the hybrid facility, plotted as functions of the hybrid penetration parameter $W_{\rm hyb}/(W^{\rm sr}_{\rm F}+W_{\rm hyb})$ for various values of $\gamma^{\rm sr}_{\rm RW}$. The other parameters that define the hybrid plant and the local area are listed in Table 1.

share of renewable electricity as the penetration of hybrid facilities increases in the local area, while the slope of $\beta^{\rm hyb}_{\rm WF}$ and $\eta^{\rm hyb}_{\rm FW}$ is positive when $\eta^{\rm hyb}_{\rm FW}>\eta^{\rm sr}_{\rm FW}$ at $x=0$. In both the upper and lower graphs in Fig. 3 the curves become flat when $\eta^{\rm hyb}_{\rm FW}$ approaches the value 0.4697 which correspond to the condition $\eta^{\rm hyb}_{\rm RW}=\eta^{\rm sr}_{\rm RW}=0.153$: in this case the SRSPR and STALPR allocation always coincide regardless of the value of x. The differences between the SRSPR and STALPR allocations become significant as $\eta^{\rm hyb}_{\rm FW}$ departs from this value and in general increase as the hybrid facilities take on a sizable fraction of the production of electricity in the local area.

Similar considerations apply to Fig. 4 which shows the sensitivity analysis carried out by varying the parameter $\eta^{\rm sr}_{\rm RW}$, within the range 0.05–0.35. It can be noted that at $x=0$ we always have the conditions $\eta^{\rm hyb}_{\rm RW}>\eta^{\rm sr}_{\rm RW}$ (upper graph) and $\eta^{\rm hyb}_{\rm FW}>\eta^{\rm sr}_{\rm FW}$ (lower graph). We obtain in this case that the slope of $\beta^{\rm hyb}_{\rm WR}$ and $\eta^{\rm hyb}_{\rm RW}$ is negative while it is positive for $\beta^{\rm hyb}_{\rm WF}$ and $\eta^{\rm hyb}_{\rm FW}$.

The effect of the variation of $\gamma^{\rm sr}_{\rm RW}$ on the allocation fraction and efficiencies is shown in Fig. 5. The slope of the curves at $x=0$ changes sign when $\gamma^{\rm sr}_{\rm RW}$ exceeds about 0.05. The effect of x on the allocation fractions and partial efficiencies are significant for values of $\gamma^{\rm sr}_{\rm RW}$ below 0.01. For instance for $\gamma^{\rm sr}_{\rm RW}=0.005$ we see from the upper graph of Fig. 5 that $\beta^{\rm hyb}_{\rm WR}$ reaches the maximum value of 0.125 at $x=0.25$ and then decreases to 0.04 at $x=1$.

The results reported in Figs. 3–5, although representative of a fairly wide range of local area conditions, are specific to the characteristics of the hybrid plant chosen as our case study (i.e., characterized by the given yearly energy balance). Clearly, by selecting a different hybrid technology with different ratio of fossil-to-renewable primary energy and efficiency, as well as by changing the conditions of its progressive penetration within the local area, would yield different results in terms of allocation fraction profiles. Even more complex situations may arise when in the same local area different hybrid plants adopt different hybridization technologies. Nonetheless, such cases can all be addressed by applying the general equations of the STLAPR method that are given in Section 3.

The simplified situation adopted in our case study and the results just discussed provide general indications on how a fair allocation method can help to identify a fair incentive policy for hybrid plants. This important issue is addressed in the next section.

5. Rationale for incentive policies related to hybrid plants

One general principle that is usually considered by incentive policies is to provide access to subsidies based on the renewable fraction of the electricity generated by the hybrid plant. In this context, the proposed SRSPR and STALPR allocation methods are useful in that they define a fair identification of the share of renewable electricity which qualifies for the subsidies. However, we believe that such a general principle, which focuses only on the share of renewable electricity, might in some cases result in supporting hybrid solutions that are unfavorable for the local area.

To better explain this concept let us consider again the allocation results of the case study presented in Section 4. By comparing the top and bottom graphs of Figs. 3–5, it can be noted that high values of $\eta^{\rm hyb}_{\rm RW}$ (or $\beta^{\rm hyb}_{\rm WR}$) correspond to low values of $\eta^{\rm hyb}_{\rm FW}$ (or $\beta^{\rm hyb}_{\rm WF}$) and viceversa, as a consequence of the constraint given in Eq. (2) and of Eqs. (14) and (15). Considering for instance the curve in Fig. 3 (top) with $\eta^{\rm sr}_{\rm FW}=0.55$, at $x=0$ we have $\eta^{\rm hyb}_{\rm RW}=0.13$ and $\beta^{\rm hyb}_{\rm WR}=0.067$. Thus, according to the general criterion, the hybrid facility should be qualified for the subsidies in proportion to the value of $\beta^{\rm hyb}_{\rm WR}$. Also, since $\beta^{\rm hyb}_{\rm WR}$ increases with x, the incentive policy would encourage the penetration of such hybrid facilities in the local area.

On the other hand, from the corresponding curve with $\eta^{sr}_{FW} = 0.55$ in Fig. 3 (bottom) at $x = 0$ we obtain $\eta^{hyb}_{\substack{F\\W}} = 0.475$, a value significantly lower than the reference efficiency of the single resource facilities of the local area. Thus, an important question arises as to whether this comparatively lower fossil conversion efficiency might overwhelm the overall advantages generally expected from hybridization.

Again, the allocation analysis provides a sound framework to address incentive policy issues, because it provides values for the partial primary energy factors $f^{hyb}_{\substack{F\\W}}$ and $f^{hyb}_{\substack{R\\W}}$. Except for the FCSS method, the assignments based on ExRR, SRSPR, and STALPR methods can all be readily expressed as follows

$$f^{hyb}_{\substack{R\\W}} = \frac{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}}} + \dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}}}}{\dfrac{W_{hyb}}{f^{x}_{\substack{R\\W}}}} \text{ and } f^{hyb}_{\substack{F\\W}} = \frac{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}}} + \dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}}}}{\dfrac{W_{hyb}}{f^{x}_{\substack{F\\W}}}} \tag{45}$$

where the ‘x’ superscript in $f^{x}_{\substack{F\\W}}$ and $f^{x}_{\substack{R\\W}}$ stands for ‘Ex’, ‘ref’, and ‘loc’, respectively.

The following parameters characterize from different points of view a hybrid facility and are all well defined once an allocation method has been chosen and solved:

$$\mathrm{PFES_{hyb}} = 1 - \frac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}} W^{hyb}_{F}} = 1 - \frac{f^{hyb}_{\substack{F\\W}}}{f^{x}_{\substack{F\\W}}} = 1 - \frac{\eta^{x}_{FW}}{\eta^{hyb}_{\substack{F\\W}}} \tag{46}$$

$$\mathrm{PRES_{hyb}} = 1 - \frac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}} W^{hyb}_{R}} = 1 - \frac{f^{hyb}_{\substack{R\\W}}}{f^{x}_{\substack{R\\W}}} = 1 - \frac{\eta^{x}_{RW}}{\eta^{hyb}_{\substack{R\\W}}} \tag{47}$$

$$\mathrm{IFEI_{hyb}} = \frac{W^{hyb}_{F}}{\dfrac{P^{hyb}_{F}}{f^{x}_{\substack{F\\W}}}} - 1 = \frac{f^{x}_{\substack{F\\W}}}{f^{hyb}_{\substack{F\\W}}} - 1 = \frac{\eta^{hyb}_{\substack{F\\W}}}{\eta^{x}_{FW}} - 1 \tag{48}$$

$$\mathrm{IREI_{hyb}} = \frac{W^{hyb}_{R}}{\dfrac{P^{hyb}_{R}}{f^{x}_{\substack{R\\W}}}} - 1 = \frac{f^{x}_{\substack{R\\W}}}{f^{hyb}_{\substack{R\\W}}} - 1 = \frac{\eta^{hyb}_{\substack{R\\W}}}{\eta^{x}_{RW}} - 1 \tag{49}$$

where the ‘x’ superscript in $f^{x}_{\substack{F\\W}}$ and $f^{x}_{\substack{R\\W}}$ stands for ‘ref’, ‘Ex’, ‘ref’, or ‘loc’ depending on whether the allocation method used is FCSS, ExRR, SRSPR, and STALPR, respectively.

$\mathrm{PFES_{hyb}}$ (Primary Fossil Energy Savings of the hybrid facility) expresses the savings of primary fossil energy that the hybrid facility achieves while producing the same amount of electricity allocated to the fossil source in a single-resource facility operating with the reference primary energy factor $f^{x}_{\substack{F\\W}}$ adopted as basis for the allocation, and hence with the efficiency $\eta^{x}_{FW} = f^{hyb}_{F}/f^{x}_{\substack{F\\W}}$.

The same concept is expressed by $\mathrm{PRES_{hyb}}$ (Primary Renewable Energy Savings of the hybrid facility) which expresses the savings of renewable primary energy that the hybrid facility attains while producing the same amount of electricity allocated to the renewable source in a single-resource reference facility operating with the reference primary energy factor $f^{x}_{\substack{R\\W}}$ adopted as basis for the allocation, and hence with the efficiency $\eta^{x}_{RW} = f^{hyb}_{R}/f^{x}_{\substack{R\\W}}$.

$\mathrm{IFEI_{hyb}}$ (Incremental Fossil Electricity Index of the hybrid facility) expresses the increment in electricity allocated to the fossil source with respect to the electricity that can be produced with the same amount of fossil primary energy used by the hybrid facility in a single-resource reference facility operating with the reference primary energy factor $f^{x}_{\substack{F\\W}}$ adopted as basis for the allocation, and hence with the efficiency $\eta^{x}_{FW} = f^{hyb}_{F}/f^{x}_{\substack{F\\W}}$.

$\mathrm{IREI_{hyb}}$ (Incremental Renewable Electricity Index of the hybrid facility) expresses the increment in electricity allocated to the renewable source with respect to the electricity that can be produced with the same amount of renewable primary energy used by the hybrid facility, in single-resource reference facilities operating with the reference primary energy factor $f^{x}_{\substack{R\\W}}$ adopted as basis for the allocation, and hence with the efficiency $\eta^{x}_{RW} = f^{hyb}_{R}/f^{x}_{\substack{R\\W}}$.

It should be noted from Eqs. (46)–(49) that the signs of $\mathrm{PFES_{hyb}}$ and $\mathrm{IFEI_{hyb}}$, and of $\mathrm{PRES_{hyb}}$ and $\mathrm{IREI_{hyb}}$ are all positive whenever $\eta^{hyb}_{\substack{F\\W}} > \eta^{x}_{\substack{F\\W}}$ and $\eta^{hyb}_{\substack{R\\W}} > \eta^{x}_{\substack{R\\W}}$, respectively. Moreover, they are clearly interrelated as follows

$$1 + \mathrm{IFEI_{hyb}} = \frac{1}{1 - \mathrm{PFES_{hyb}}} = \frac{\eta^{hyb}_{\substack{F\\W}}}{\eta^{x}_{FW}} \tag{50}$$

$$1 + \mathrm{IREI_{hyb}} = \frac{1}{1 - \mathrm{PRES_{hyb}}} = \frac{\eta^{hyb}_{\substack{R\\W}}}{\eta^{x}_{RW}} \tag{51}$$

Because of their simple and direct relation to $\eta^{hyb}_{\substack{F\\W}}$ and $\eta^{hyb}_{\substack{R\\W}}$, the profiles of $\mathrm{PFES_{hyb}}$. $\mathrm{IFEI_{hyb}}$, $\mathrm{PRES_{hyb}}$, and $\mathrm{IREI_{hyb}}$ for the case study defined in Section 4, can be readily derived and almost read out by inspection from the profiles reported in Figs. 3–5.

Incentive policies intended to focus on the fact that fossil-solar hybridization may result in a more efficient utilization of fossil fuels, should promote positive values of $\mathrm{PFES_{hyb}}$ or, equivalently, of $\mathrm{IFEI_{hyb}}$, calculated with respect to a reference or threshold value of $\eta^{x}_{\substack{F\\W}}$ which is either set by some authority, $\eta^{ref}_{\substack{F\\W}}$, or is the local average value $\eta^{loc}_{\substack{F\\W}}$ defined by the STALPR method.

Incentive policies intended to focus on the fact that fossil-solar hybridization may result in an efficient utilization of renewable resources, should promote positive values of $\mathrm{PRES_{hyb}}$ or, equivalently, of $\mathrm{IREI_{hyb}}$, calculated with respect to a reference or threshold value of $\eta^{x}_{\substack{R\\W}}$ which is either set by some authority,

$\eta^{\mathrm{ref}}_{\mathrm{R}_\mathrm{W}}$, or is the local average value $\eta^{\mathrm{loc}}_{\mathrm{R}_\mathrm{W}}$ defined by the STALPR method, and it is important that this will vary from one renewable resource to another (e.g., $\eta^{\mathrm{ref}}_{\mathrm{R}_\mathrm{W}}$ for solar radiation will be generally different from $\eta^{\mathrm{ref}}_{\mathrm{R}_\mathrm{W}}$ for biomass or for geothermal energy).

It is also possible to define a single parameter that combines both aspects with the idea to promote only facilities that are incremental in both aspects. For example one could define the overall Primary Energy Savings (PES) coefficient

$$\mathrm{PES}_{\mathrm{hyb}} = 1 - \frac{P^{\mathrm{hyb}}_{\mathrm{R}} + P^{\mathrm{hyb}}_{\mathrm{F}}}{f^{\mathrm{x}}_{\mathrm{R}_\mathrm{W}} W^{\mathrm{hyb}}_{\mathrm{R}} + f^{\mathrm{x}}_{\mathrm{F}_\mathrm{W}} W^{\mathrm{hyb}}_{\mathrm{F}}} = 1 - \frac{\dfrac{f_{\mathrm{hyb}}}{\eta_{\mathrm{hyb}}}}{\beta^{\mathrm{hyb}}_{\mathrm{W}_\mathrm{R}} \dfrac{f^{\mathrm{hyb}}_{\mathrm{R}}}{\eta^{\mathrm{x}}_{\mathrm{RW}}} + \left(1 - \beta^{\mathrm{hyb}}_{\mathrm{W}_\mathrm{R}}\right) \dfrac{f^{\mathrm{hyb}}_{\mathrm{F}}}{\eta^{\mathrm{x}}_{\mathrm{FW}}}} \tag{52}$$

However it should be noted that this parameter is conceptually improper in that it sums on equal grounds primary fuel energy and renewable energy in spite of their different physical and economical values. The $\mathrm{PES}_{\mathrm{hyb}}$ coefficient expresses the overall primary energy savings (both fossil and renewable) that the hybrid facility attains with respect to the production of the same electricity, in two distinct single-resource reference facilities with $f^{\mathrm{x}}_{\mathrm{R}_\mathrm{W}} = f^{\mathrm{hyb}}_{\mathrm{R}}/\eta^{\mathrm{x}}_{\mathrm{RW}}$ and $f^{\mathrm{x}}_{\mathrm{F}_\mathrm{W}} = f^{\mathrm{hyb}}_{\mathrm{F}}/\eta^{\mathrm{x}}_{\mathrm{FW}}$. In general, i.e., regardless of the allocation method used to compute $f^{\mathrm{hyb}}_{\mathrm{F}_\mathrm{W}}$ and $f^{\mathrm{hyb}}_{\mathrm{R}_\mathrm{W}}$, it is related to $\mathrm{PFES}_{\mathrm{hyb}}$ and $\mathrm{PRES}_{\mathrm{hyb}}$ as follows

$$\frac{1 + \sigma_{\mathrm{hyb}}}{1 - \mathrm{PES}_{\mathrm{hyb}}} = \frac{1}{1 - \mathrm{PFES}_{\mathrm{hyb}}} + \frac{\sigma_{\mathrm{hyb}}}{1 - \mathrm{PRES}_{\mathrm{hyb}}} \tag{53}$$

Similarly, $\mathrm{IFEI}_{\mathrm{hyb}}$ and $\mathrm{IREI}_{\mathrm{hyb}}$ are related the parameter $\mathrm{IEI}_{\mathrm{hyb}}$ defined in Eq. (37) as follows

$$\mathrm{IEI}_{\mathrm{hyb}} = \frac{\mathrm{IFEI}_{\mathrm{hyb}}\sigma_{\mathrm{hyb}} + \mathrm{IREI}_{\mathrm{hyb}}\chi^{\mathrm{x}}_{\mathrm{W}}}{\sigma_{\mathrm{hyb}} + \chi^{\mathrm{x}}_{\mathrm{W}}} \tag{54}$$

where by inspection of Eq. (37) we note that (differently from $\mathrm{PES}_{\mathrm{hyb}}$) the parameter $\mathrm{IEI}_{\mathrm{hyb}}$ is independent of the allocation problem.

It is important to note that whether we use the ExRR, the SRSPR or the STALPR allocation method (in other words, except for the FCSS method) Eqs. (30) or (36), respectively, imply the identities

$$\mathrm{PFES}_{\mathrm{hyb}} = \mathrm{PRES}_{\mathrm{hyb}} = \mathrm{PES}_{\mathrm{hyb}} = 1 - \frac{\dfrac{P^{\mathrm{hyb}}_{\mathrm{R}}}{f^{\mathrm{x}}_{\mathrm{R}_\mathrm{W}}} + \dfrac{P^{\mathrm{hyb}}_{\mathrm{F}}}{f^{\mathrm{x}}_{\mathrm{F}_\mathrm{W}}}}{W_{\mathrm{hyb}}} = \frac{\mathrm{IEI}_{\mathrm{hyb}}}{1 + \mathrm{IEI}_{\mathrm{hyb}}} \tag{55}$$

$$\mathrm{IFEI}_{\mathrm{hyb}} = \mathrm{IREI}_{\mathrm{hyb}} = \mathrm{IEI}_{\mathrm{hyb}} = \frac{W_{\mathrm{hyb}}}{\dfrac{P^{\mathrm{hyb}}_{\mathrm{R}}}{f^{\mathrm{x}}_{\mathrm{R}_\mathrm{W}}} + \dfrac{P^{\mathrm{hyb}}_{\mathrm{F}}}{f^{\mathrm{x}}_{\mathrm{F}_\mathrm{W}}}} - 1 = \frac{\mathrm{PES}_{\mathrm{hyb}}}{1 - \mathrm{PES}_{\mathrm{hyb}}} \tag{56}$$

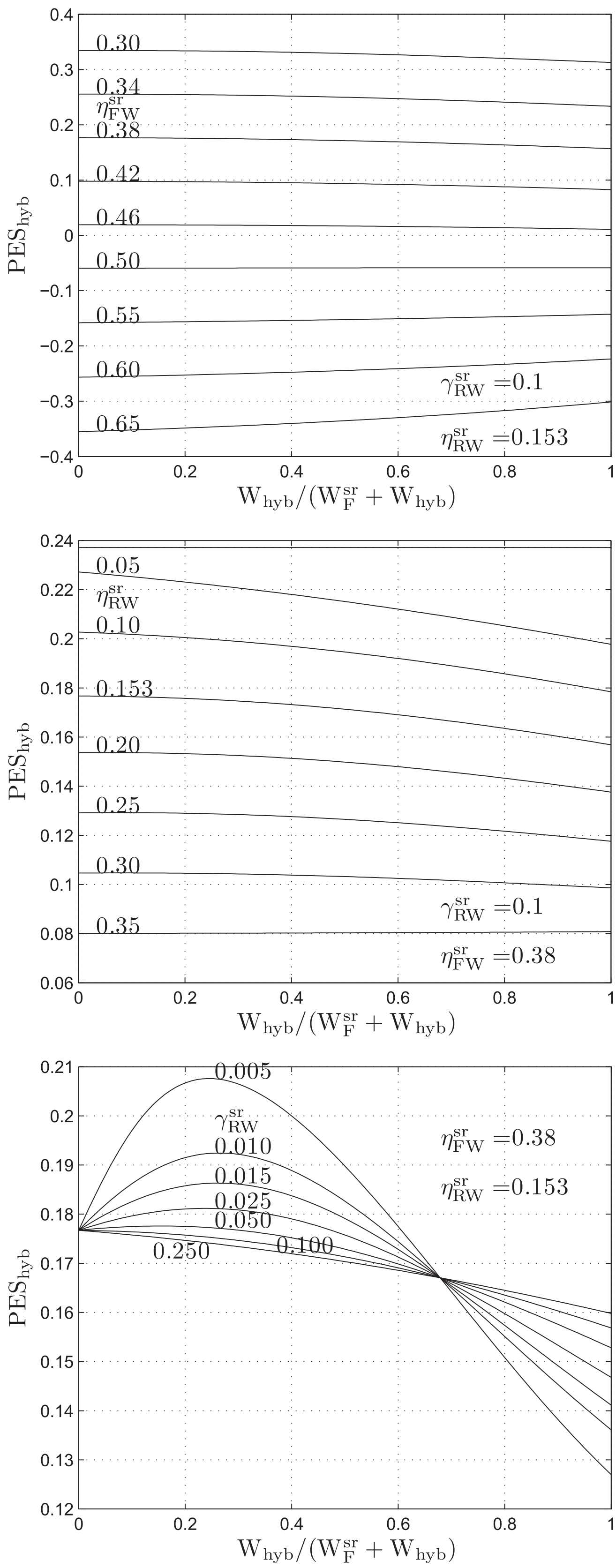

Fig. 6. Primary Energy Savings (PES) of the hybrid facility, plotted as a function of the hybrid penetration parameter $W_{\mathrm{hyb}}/(W^{\mathrm{sr}}_{\mathrm{F}} + W_{\mathrm{hyb}})$ for various values of $\eta^{\mathrm{sr}}_{\mathrm{FW}}$, $\eta^{\mathrm{sr}}_{\mathrm{RW}}$ and $\gamma^{\mathrm{sr}}_{\mathrm{RW}}$. The other parameters that define the hybrid plant and the local area are listed in Table 1.

The first equality in each of these identities proves that the ExRR, the SRSPR and the STALPR distribute the benefits of hybridization (whether measured in terms of primary energy savings or of incremental work) evenly among the two resources. For the STALPR method, the values of $f^{\mathrm{loc}}_{\mathrm{F}\atop \mathrm{W}}$ and $f^{\mathrm{loc}}_{\mathrm{R}\atop \mathrm{W}}$ must first be computed.

From the point of view of the incentives, an important threshold is when the hybrid plant produces the same electricity as the reference single-resource plant that uses the same amount of fossil fuel. Clearly in this case the solar integration results in additional costs and land use with no advantages in terms of additional electricity output, a situation which should clearly not qualify for incentives with respect to single-resource reference power plants. It is defined by the condition $W^{\mathrm{hyb}} = P^{\mathrm{hyb}}_{\mathrm{F}}/f^{\mathrm{ref}}_{\mathrm{F}\atop \mathrm{W}}$, which substituted in Eq. (55) after a few rearrangements yields the corresponding threshold (negative) value of the PES and IEI coefficients

$$\mathrm{PES}_{\mathrm{hyb}} = -\frac{P^{\mathrm{hyb}}_{\mathrm{R}}}{f^{\mathrm{ref/loc}}_{\mathrm{R}\atop \mathrm{W}} W_{\mathrm{hyb}}} \quad \text{and} \quad \mathrm{IEI}_{\mathrm{hyb}} = -\frac{P^{\mathrm{hyb}}_{\mathrm{R}}}{P^{\mathrm{hyb}}_{\mathrm{R}} + f^{\mathrm{ref/loc}}_{\mathrm{R}\atop \mathrm{W}} W_{\mathrm{hyb}}} \tag{57}$$

Below this threshold the regulator must discourage hybridization.

Fig. 6 shows the $\mathrm{PES}_{\mathrm{hyb}}$ profiles calculated at the same conditions of the sensitivity analysis of the case study carried out in Section 4. In particular, with reference to the example discussed earlier in this section, it can be noted that the curve with $\eta^{\mathrm{sr}}_{\mathrm{FW}} = 0.55$ in Fig. 6 (obtained with the same assumptions of the corresponding curves in Figs. 3–5) results in negative values of $\mathrm{PES}_{\mathrm{hyb}}$, thus denoting an overall lower conversion efficiency of the hybrid plant with respect to the single resource facilities.

6. Conclusions

In several industrial and manufacturing sectors, higher production efficiencies are achieved by integrating the production of a mix of different products and/or using a mix of different resources and/or raw materials. Such facilities are therefore either multi-resource or multi-generation, or both. In all these cases, one key problem is how to define a 'fair' method to allocate the consumption of each resource used in the facility among the different co-generated products. This is for instance the case of multi-product systems typically addressed in life cycle analysis problems.

Among the possible combinations between resources and products, one case of particular interest for the energy sector is that of cogeneration facilities, where the objective of the allocation problem is to determine the fraction of fuel consumption in the cogenerator that should be assigned to the production of heat and that to the production of electrical energy. We address this case in a previous paper [1], motivated by the need to overcome the limitations of the classical allocation methods which require some prescribed reference primary energy conversion efficiencies defined by some authority. In Ref. [1], we propose a self-consistent method that we call STLAPR whereby the allocation is adaptive and self-tuned to the local energy scenario, with no need for prescribed reference efficiencies.

In this paper, we apply the same overall logic to formulate the STLARP method for the allocation problem of the hybrid power production facilities, based on the integration of renewable resources into conventional fossil-fuel-fired power plants. In particular, we focus on the production of electricity in hybrid fossil + solar power plants, because they represent a desirable solution in terms of a reliable use of the solar energy, capable of mitigating its intrinsic intermittency. The goal of the allocation problem is to determine what fraction of the electricity is to be considered as produced from the solar energy and therefore qualifies for the subsidies typically provided to promote the uses of renewable energy.

The results of our analysis are demonstrated using a realistic case study, where we consider a scenario in which Solar-Integrated Combined-Cycle Systems (SICCSs) progressively replace the electricity produced by fossil-only facilities in an area where both fossil-only and renewable-only power plant were the norm. An analytical study of the equations governing the STLARP method reveals the key parameters of the local area that determine the electricity allocation fractions as functions of the penetration of the SICCS plants in the local area. The differences between the SRSPR method based on prescribed reference efficiencies and the proposed method turns out to become significant as the hybrid facilities produce a sizable fraction of the production of electricity in the local area.

For illustrative purposes we considered a basic case with a single hybrid plant in the local area. Even in such case, computing the STALRP allocation is mathematically less straightforward than computing the SRSPR allocation. It requires the solution of a nonlinear system of ten equations in the unknowns, which nevertheless we show in Appendix A can be reduced to a much simpler single second-degree equation. However, as other hybrid plants are added to the local area the higher number of equations increases the complexity of the solution. However, we note that (1) such mathematical complexity can be easily and effectively handled nowadays by numerical algorithms available in ordinary solvers; and (2) a simplified version of the STLARP method may also be obtained by updating from time to time the reference values of the SRSPR method, according to the evolution of the energy portfolio of the local area. This approach may be easily implemented into regulations and would maintain the simplicity of the SRSPR method, but requires a frequent update of the reference values describing the effective efficiency of the local area.

Although this paper focuses on the specific allocation problem of a hybrid power plant, the principle of fair allocation and the implementation scheme we provide here applies in general to a wide variety of multi-resource multi-product systems, and not only in the energy sector. In a forthcoming paper, we further generalize the formulation of the SRSPR and the STALPR allocation methods to the case of hybrid cogeneration facilities (two resources and two products) as well as multi-resource multi-generation facilities. The natural extension of the case studies of the present work and Ref. 1, is indeed the study of the penetration in local area scenario of hybrid solar-integrated heat (or desalination) and power cogeneration facilities to replace non-hybrid fossil-fuel-operated separate productions of heat and electricity.

Acknowledgment

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Appendix A

First, we define the following ratio

$$\sigma_{\mathrm{loc}} = \frac{P_{\mathrm{F}}^{\mathrm{sr}} + P_{\mathrm{F}}^{\mathrm{hyb}}}{P_{\mathrm{R}}^{\mathrm{sr}} + P_{\mathrm{R}}^{\mathrm{hyb}}} \tag{A1}$$

and we write Eqs. (9) and (10) corresponding to the closure condition (35)

$$\beta_{\mathrm{W}\,\mathrm{R}}^{\mathrm{hyb}} = \frac{\chi_{\mathrm{W}}^{\mathrm{loc}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{loc}}} \quad \text{and} \quad \beta_{\mathrm{W}\,\mathrm{F}}^{\mathrm{hyb}} = \frac{\sigma_{\mathrm{hyb}}}{\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{loc}}} \tag{A2}$$

$$f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{hyb}} = \frac{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{loc}}\right) f_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + 1\right) \chi_{\mathrm{W}}^{\mathrm{loc}} \eta_{\mathrm{hyb}}} \quad \text{and} \quad f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{hyb}} = \frac{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{loc}}\right) f_{\mathrm{hyb}}}{\left(\sigma_{hyb} + 1\right) \eta_{\mathrm{hyb}}} \tag{A3}$$

Next, by defining the fractions of the fossil energy and renewable energy consumed by the hybrid plant in the local area

$$\begin{aligned} \rho_{\mathrm{R}}^{\mathrm{hyb}} &= 1 - \rho_{\mathrm{R}}^{\mathrm{sr}} = \frac{P_{\mathrm{R}}^{\mathrm{hyb}}}{P_{\mathrm{R}}^{\mathrm{sr}} + P_{\mathrm{R}}^{\mathrm{hyb}}} \quad \text{and} \\ \rho_{\mathrm{F}}^{\mathrm{hyb}} &= 1 - \rho_{\mathrm{F}}^{\mathrm{sr}} = \frac{P_{\mathrm{F}}^{\mathrm{hyb}}}{P_{\mathrm{F}}^{\mathrm{sr}} + P_{\mathrm{F}}^{\mathrm{hyb}}} = \frac{\sigma_{\mathrm{hyb}}}{\sigma_{\mathrm{loc}}} \rho_{\mathrm{R}}^{\mathrm{hyb}} \end{aligned} \tag{A4}$$

the average primary energy factors for the fossil fuel and the renewable source conversion to electricity in the local area can be expressed as

$$f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{loc}} = \left(\frac{\rho_{\mathrm{R}}^{\mathrm{hyb}}}{f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{hyb}}} + \frac{1 - \rho_{\mathrm{R}}^{\mathrm{hyb}}}{f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{sr}}} \right)^{-1} \quad \text{and} \quad f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{loc}} = \left(\frac{\rho_{\mathrm{F}}^{\mathrm{hyb}}}{f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{hyb}}} + \frac{1 - \rho_{\mathrm{F}}^{\mathrm{hyb}}}{f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}}} \right)^{-1} \tag{A5}$$

Taking the ratio of the two equations (A5) to compute χ_{loc} according to Eq. (35) and using Eq. (A3) to eliminate $f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{hyb}}$ and $f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{hyb}}$ we obtain the following relation

$$\chi_{\mathrm{W}}^{\mathrm{loc}} = \frac{\dfrac{1 - \rho_{\mathrm{R}}^{\mathrm{hyb}}}{f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{sr}}} + \rho_{\mathrm{R}}^{\mathrm{hyb}} \dfrac{\left(\sigma_{\mathrm{hyb}} + 1\right) \eta^{\mathrm{hyb}} \chi_{\mathrm{loc}}}{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{loc}}\right) f_{\mathrm{hyb}}}}{\dfrac{1 - \rho_{\mathrm{F}}^{\mathrm{hyb}}}{f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}}} + \rho_{\mathrm{F}}^{\mathrm{hyb}} \dfrac{\left(\sigma_{\mathrm{hyb}} + 1\right) \eta_{\mathrm{hyb}}}{\left(\sigma_{\mathrm{hyb}} + \chi_{\mathrm{W}}^{\mathrm{loc}}\right) f_{\mathrm{hyb}}}} \tag{A6}$$

This equation defines $\chi_{\mathrm{W}}^{\mathrm{loc}}$ implicitly in terms of the parameters σ_{hyb}, η_{hyb}, f_{hyb} of the hybrid plant and the local parameters σ_{loc}, $\rho_{\mathrm{R}}^{\mathrm{hyb}}$, $f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{sr}}$ and $f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}}$. With a few rearrangements and using the last of (A4), Eq. (A6) can be finally cast as:

$$\begin{aligned} &\left(\sigma_{\mathrm{loc}} - \rho_{\mathrm{R}}^{\mathrm{hyb}} \sigma_{\mathrm{hyb}}\right) f_{\mathrm{hyb}} f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{sr}} \left(\chi_{\mathrm{W}}^{\mathrm{loc}}\right)^2 \\ &+ \Big[f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{sr}} f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}} \eta_{\mathrm{hyb}} \rho_{\mathrm{R}}^{\mathrm{hyb}} \left(\sigma_{\mathrm{hyb}} - \sigma_{\mathrm{loc}}\right) \left(\sigma_{\mathrm{hyb}} + 1\right) \\ &\quad + \left(\sigma_{\mathrm{loc}} - \rho_{\mathrm{R}}^{\mathrm{hyb}} \sigma_{\mathrm{hyb}}\right) \sigma_{\mathrm{hyb}} f_{\mathrm{hyb}} f_{\mathrm{R}\,\mathrm{W}}^{\mathrm{sr}} - \left(1 - \rho_{\mathrm{R}}^{\mathrm{hyb}}\right) \sigma_{\mathrm{loc}} f_{\mathrm{hyb}} f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}} \Big] \chi_{\mathrm{W}}^{\mathrm{loc}} \\ &- \left(1 - \rho_{\mathrm{R}}^{\mathrm{hyb}}\right) \sigma_{\mathrm{hyb}} \sigma_{\mathrm{loc}} f_{\mathrm{hyb}} f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}} = 0 \end{aligned} \tag{A7}$$

This second-order equation in $\chi_{\mathrm{W}}^{\mathrm{loc}}$ can be easily solved for the only positive root it admits.[9] The product allocation fractions and the primary energy factors can then be readily obtained from Eqs. (A2) and (A3).

Appendix B

There is a direct duality between the problem we treat in Appendix A of the present paper leading to the second-order Eq. (A7) and the problem we treat in Section 3 of Ref. [1] leading to the second-order Eq. (30) of Ref. [1] which we copy here for convenience

$$\begin{aligned} &\left(\sigma_{\mathrm{loc}} - \gamma_{\mathrm{Q,chp}} \sigma_{\mathrm{chp}}\right) \eta_{\mathrm{chp}} \overline{f}_{\mathrm{el,sep}} \Phi_{\mathrm{loc}}^2 + \Big[f_{\mathrm{F,chp}} \gamma_{\mathrm{Q,chp}} \left(\sigma_{\mathrm{chp}} - \sigma_{\mathrm{loc}}\right) \\ &\times \left(\sigma_{\mathrm{chp}} + 1\right) + \left(\sigma_{\mathrm{loc}} - \gamma_{\mathrm{Q,chp}} \sigma_{\mathrm{chp}}\right) \sigma_{\mathrm{chp}} \eta_{\mathrm{chp}} \overline{f}_{\mathrm{el,sep}} \\ &- \left(1 - \gamma_{\mathrm{Q,chp}}\right) \sigma_{\mathrm{loc}} \eta_{\mathrm{chp}} \overline{f}_{\mathrm{Q,sep}} \Big] \Phi_{\mathrm{loc}} - \left(1 - \gamma_{\mathrm{Q,chp}}\right) \\ &\times \sigma_{\mathrm{chp}} \sigma_{\mathrm{loc}} \eta_{\mathrm{chp}} \overline{f}_{\mathrm{Q,sep}} = 0 \end{aligned} \tag{30[1]}$$

In fact, with the help of Fig. B1, it is easy to verify that these two equations, as well as Eq. (A3) and (25 [1]) and all the others, convert into one another by the substitutions detailed in Table B1. As a result, the analysis of Eq. (30 [1]) in the Appendix of Ref. [1] applies also here with the substitutions listed in Table B1, where we observe that the parameter $\mathrm{IEI_{hyb}}$ defined in Eq. (37) may be written as

$$\mathrm{IEI_{hyb}} = \frac{\eta_{\mathrm{hyb}}}{f_{\mathrm{hyb}}} f_{\mathrm{F}\,\mathrm{W}}^{\mathrm{sr}} \frac{cd + 1}{(c+1)d} - 1$$

[9] To see this, rewrite Eq. (A7) as $Ay^2 + By - C = 0$ where $A \geq 0$ and $C \geq 0$ and, therefore, $\sqrt{B^2 + 4AC} > B$ and $\chi_{\mathrm{W}}^{\mathrm{loc}} = (-B + \sqrt{B^2 + 4AC})/2A$ is the only positive root. That $C \geq 0$ is trivial. That $A \geq 0$ can be readily seen after noting that $\sigma_{\mathrm{loc}} - \rho_{\mathrm{R}}^{\mathrm{hyb}} \sigma_{\mathrm{hyb}} = P_{\mathrm{R}}^{\mathrm{sr}} / (P_{\mathrm{R}}^{\mathrm{sr}} + P_{\mathrm{R}}^{\mathrm{hyb}})$.

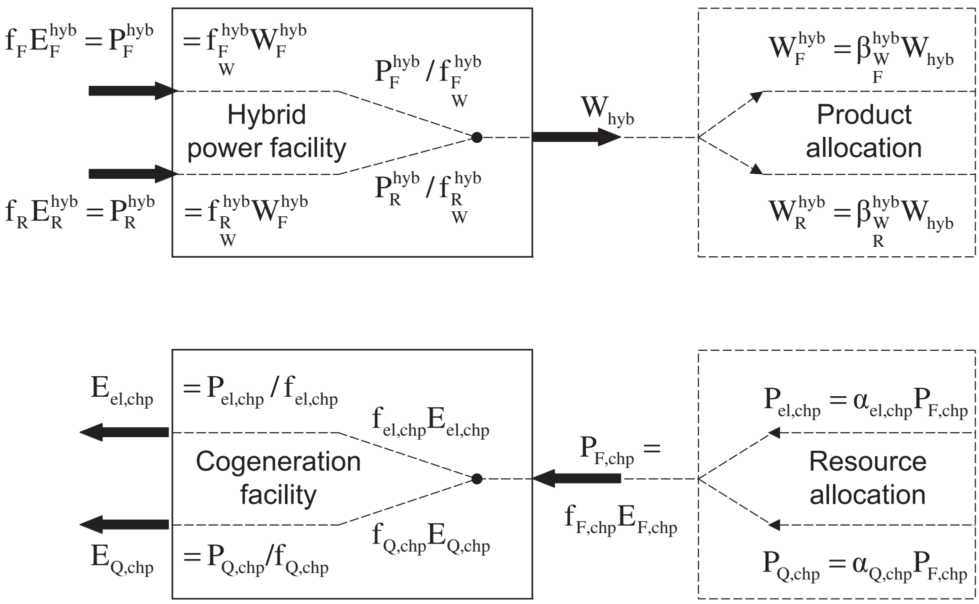

Fig. B1. Duality between the product allocation problem in a single-product hybrid power facility treated in Section 3 and Appendix A of the present paper and the conjugate resource allocation problem in a single-resource cogeneration facility treated in Section 3 and the Appendix of Ref. [1].

Table B1
Substitutions of symbols that transform the allocation problem treated in Section 3 and Appendix A of the present paper into the conjugate allocation problem treated in Section 3 and the Appendix of Ref. [1].

Ref. [1] symbols ⇔ present paper	Ref. [1] symbols ⇔ present paper	Ref. [1] symbols ⇔ present paper
$f_{F,chp} E_{F,chp} \Leftrightarrow W_{hyb}$	$f_{F,chp}/\eta_{chp} \Leftrightarrow \eta_{hyb}/f_{hyb}$	$\eta^*/\eta_{chp} \Leftrightarrow 1 + \mathrm{IEI}_{hyb}$
$E_{Q,chp} \Leftrightarrow P_R^{hyb} = f_R^{hyb} E_R^{hyb}$	$\sum_{i=1}^{m} E_{Q,sep,i} \Leftrightarrow P_R^{sr} = f_R^{sr,R} E_R^{sr}$	$\sigma_{loc} \Leftrightarrow \sigma_{loc}$
$E_{el,chp} \Leftrightarrow P_F^{hyb} = f_F^{hyb} E_F^{hyb}$	$\sum_{i=1}^{n} E_{el,sep,i} \Leftrightarrow P_F^{sr} = f_F^{sr,F} E_F^{sr}$	$\sigma_{chp} \Leftrightarrow \sigma_{hyb}$
$f_{Q,chp} \Leftrightarrow \frac{1}{f_{\substack{R\\W}}^{hyb}}$	$\bar{f}_{Q,sep} \Leftrightarrow \frac{1}{f_{\substack{R\\W}}^{sr}}$	$a = 1 - \frac{\sigma_{chp}}{\sigma_{loc}} \Leftrightarrow a = 1 - \frac{\sigma_{hyb}}{\sigma_{loc}}$
$f_{el,chp} \Leftrightarrow \frac{1}{f_{\substack{F\\W}}^{hyb}}$	$\bar{f}_{el,sep} \Leftrightarrow \frac{1}{f_{\substack{F\\W}}^{sr}}$	$b = 1 - \eta^*/\eta_{chp} \Leftrightarrow b = -\mathrm{IEI}_{hyb}$
$\alpha_{Q,chp} \Leftrightarrow \beta_{\substack{W\\R}}^{hyb}$	$\gamma_{Q,chp} \Leftrightarrow \rho_R^{hyb}$	$c = \sigma_{chp}/d \Leftrightarrow c = \sigma_{hyb}/d$
$\alpha_{el,chp} \Leftrightarrow \beta_{\substack{W\\F}}^{hyb}$	$\gamma_{el,chp} \Leftrightarrow \rho_F^{hyb}$	$d = \frac{\bar{f}_{Q,sep}}{\bar{f}_{el,sep}} \Leftrightarrow d = \frac{f_{\substack{F\\W}}^{sr}}{f_{\substack{R\\W}}^{sr}}$
$x = \gamma_{Q,chp} \Leftrightarrow x = \rho_R^{hyb}$	$\Phi_{loc} \Leftrightarrow \chi_W^{loc}$	$y = \Phi_{loc}/d \Leftrightarrow y = \frac{\chi_W^{loc}}{d}$

Nomenclature

E	energy
Ex	exergy
f	primary energy factor
$\bar{f}$	average primary energy factor
P	primary energy
Q	heat
W	electricity

Superscripts

Ex	exergy
hyb	hybrid
loc	local area of interest
ref	reference
sr	single resource

Subscripts

F	fossil
hyb	hybrid
sr	single resource
R	renewable

Greek symbols

β	allocation fraction
γ_{RW}^{sr}	fraction of the overall electricity produced in the local area that comes form the renewable-only facilities
η_{hyb}	overall efficiency of the chp plant defined by Eq. (7)
$\eta_{\substack{i\\j}}^k$ and η_{ij}^k	partial energy-conversion efficiency of facility k from resource i to product j
ρ_i^k	fractions of resource i in the local area consumed by facility k, defined by Eq. (A4)

σ_{hyb} nondimensional parameter defined by Eq. (6)
σ_{loc} nondimensional parameter defined by Eq. (A1)
χ_j^i nondimensional parameter defined by Eqs. ((5), (8), (22) and (29)

Acronyms
ExRR Exergy-based Reversible-Reference (allocation method)
FCSS Fossil-Centered-Solar-Share (allocation method)
IEI Incremental Electricity Index
$IFEI_{hyb}$ Incremental Fossil Electricity Index of the hybrid facility
$IREI_{hyb}$ Incremental Renewable Electricity Index of the hybrid facility
LHV Lower Heating Value
$PFES_{hyb}$ Primary Fossil Energy Savings of the hybrid facility
$PRES_{hyb}$ Primary Renewable Energy Savings of the hybrid facility
SICCS Solar-Integrated Combined-Cycle System
SRSPR Single-Resource-Separate-Production-Reference (allocation method)
STALPR Self-Tuned Average-Local-Productions Reference (allocation method)

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